The Physics of Neutrinos

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Lectures :

1. Panorama of Experiments
2. Neutrino Oscillations
3. Models for Neutrino Masses
4. Neutrinos in Cosmology

Lecture III

Models for Neutrino Masses

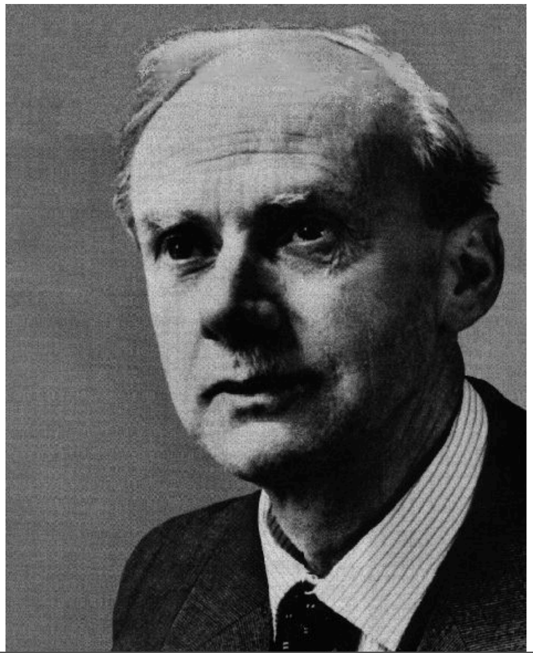
" False facts are highly injurious to the progress of science, for they often endure long; but false views, if supported by some evidence, do little harm, for every one takes a salutary pleasure in proving their falseness."

Charles Darwin

Majorana ν

×

Dirac ν



Majorana x Dirac ν

Dirac spinor

$$\Psi = P_L\Psi + P_R\Psi = \Psi_L + \Psi_R$$ 4 independent components

Dirac equation

$$i\gamma_\mu\partial^\mu\Psi_L = m\Psi_R$$

$$i\gamma_\mu\partial^\mu\Psi_R = m\Psi_L$$

Majorana x Dirac ν

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if $m = 0$

Weyl (1929)

2-component spinor is enough (Ψ_L or Ψ_R)

Pauli (1933) rejected this idea because leads to Parity Violation

Landau, Lee-Yang, Salam (1957) propose to describe the massless neutrino by a Weyl spinor ν_L

introduced in the SM in the 60's

Majorana x Dirac ν

Can we also describe a massive fermion using a 2-component spinor? (E. Majorana, 1937)

Majorana × Dirac ν

Can we also describe a massive fermion using a 2-component spinor? (E. Majorana, 1937)

$$\Psi^c = C\,\bar{\Psi}^T$$ charge conjugate field

$$(\Psi_L)^C = (\Psi^C)_R \qquad (\Psi_R)^C = (\Psi^C)_L$$

charge conjugation change chirality

$$i\gamma_\mu \partial^\mu (\Psi_L)^c = m(\Psi_R)^c \iff i\gamma_\mu \partial^\mu (\Psi_R)^c = m(\Psi_L)^c$$

$$\Psi_{L,R} \equiv \xi\,(\Psi_{R,L})^C = \xi\,C\,\bar{\Psi}_{R,L}{}^T$$

$$\xi \equiv e^{-i\alpha}$$ phase factor

Majorana x Dirac ν

Can we also describe a massive fermion using a 2-component spinor? Yes! (E. Majorana, 1937)

ξ is unphysical - can be eliminated by rephasing

Majorana Condition: $\Psi \equiv (\Psi)^C$ particle ≡ antiparticle

Majorana Field: $\Psi = \Psi_L + \Psi_R = \Psi_L + (\Psi_L)^C$

Majorana Equation: $\mathbf{i}\gamma_\mu \partial^\mu \mathbf{\Psi_L} = \mathbf{m}\,\mathbf{C}\,\overline{\mathbf{\Psi_L}}^{\mathbf{T}}$

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e.m. current vanishes $Q \equiv 0$ neutral particle

$$\overline{\mathbf{\Psi}}\gamma^\mu\mathbf{\Psi} = \overline{\mathbf{\Psi^c}}\gamma^\mu\mathbf{\Psi^c} = -\mathbf{\Psi^T}\mathbf{C}^\dagger\gamma^\mu\mathbf{C}\overline{\mathbf{\Psi}}^{\mathbf{T}} = \overline{\mathbf{\Psi}}\mathbf{C^T}\gamma^{\mu\,\mathbf{T}}\mathbf{C}^*\mathbf{\Psi} = -\overline{\mathbf{\Psi}}\gamma^\mu\mathbf{\Psi}$$

Some Properties

$$\gamma^0 \gamma^{\mu\dagger} = \gamma^\mu \gamma^0$$

$$C^T = C^\dagger = C^{-1} = -C$$

$$C^{-1} \gamma^\mu = -\gamma^{\mu T} C^{-1}$$

$$\therefore \overline{\Psi^c} = (C \gamma^0 \Psi^*)^\dagger \gamma^0 = \Psi^T \gamma^0 C^\dagger \gamma^0 = \Psi^T C$$

$$C^T \gamma^{\mu T} C^* = (-C)\gamma^{\mu T}(-C^{-1}) = C \gamma^{\mu T} C^{-1}$$

$$\therefore C^T \gamma^{\mu T} C^* = -C C^{-1} \gamma^\mu = -\gamma^\mu$$

Majorana × Dirac ν

Dirac:

$$\nu(\vec{p}, h) \overset{\hat{P}}{\Rightarrow} \nu(-\vec{p}, -h) \overset{\hat{C}}{\Rightarrow} \bar{\nu}(-\vec{p}, -h) \overset{\hat{T}}{\Rightarrow} \bar{\nu}(\vec{p}, -h)$$

LH neutrino ($h = -1$) RH antineutrino ($h = +1$)

Majorana:

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LH neutrino ($h = -1$) RH neutrino ($h = +1$)

interactions involve on LH fields

Dirac

ν_L < destroys LH neutrino / creates RH antineutrino

$\bar{\nu}_L$ < destroys RH antineutrino / creates LH neutrino

Majorana x Dirac ν

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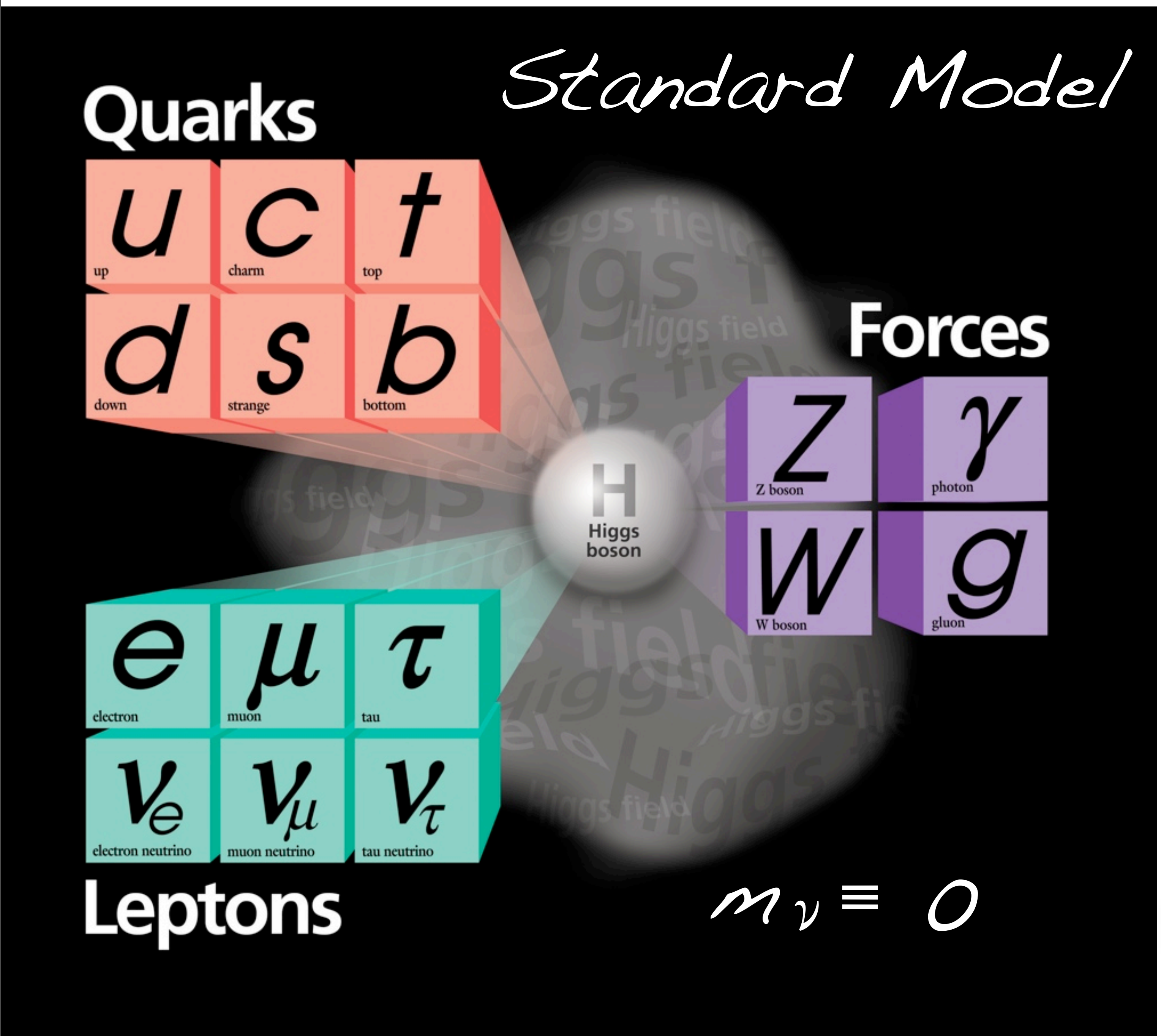
interactions involve on LH fields

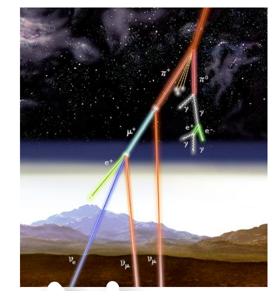
ν_L ⟨ destroys LH neutrino / creates RH neutrino

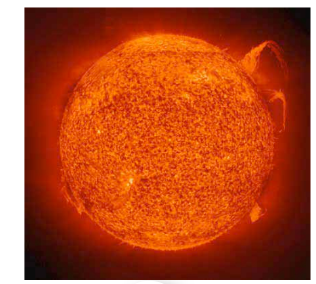
$\bar{\nu}_L$ ⟨ destroys RH neutrino / creates LH neutrino

Majorana

ν Mass Problem





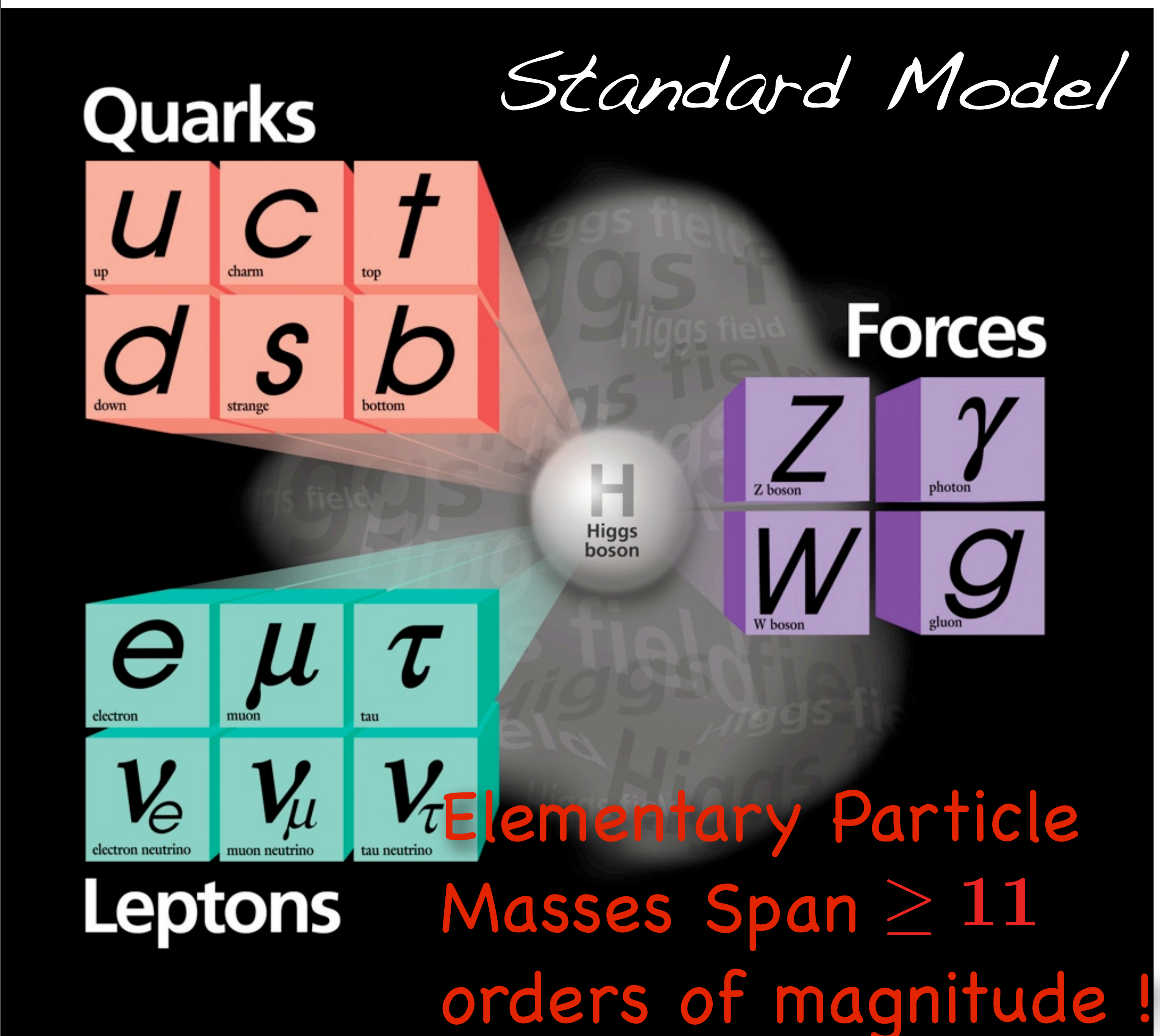


Neutrino Oscillations

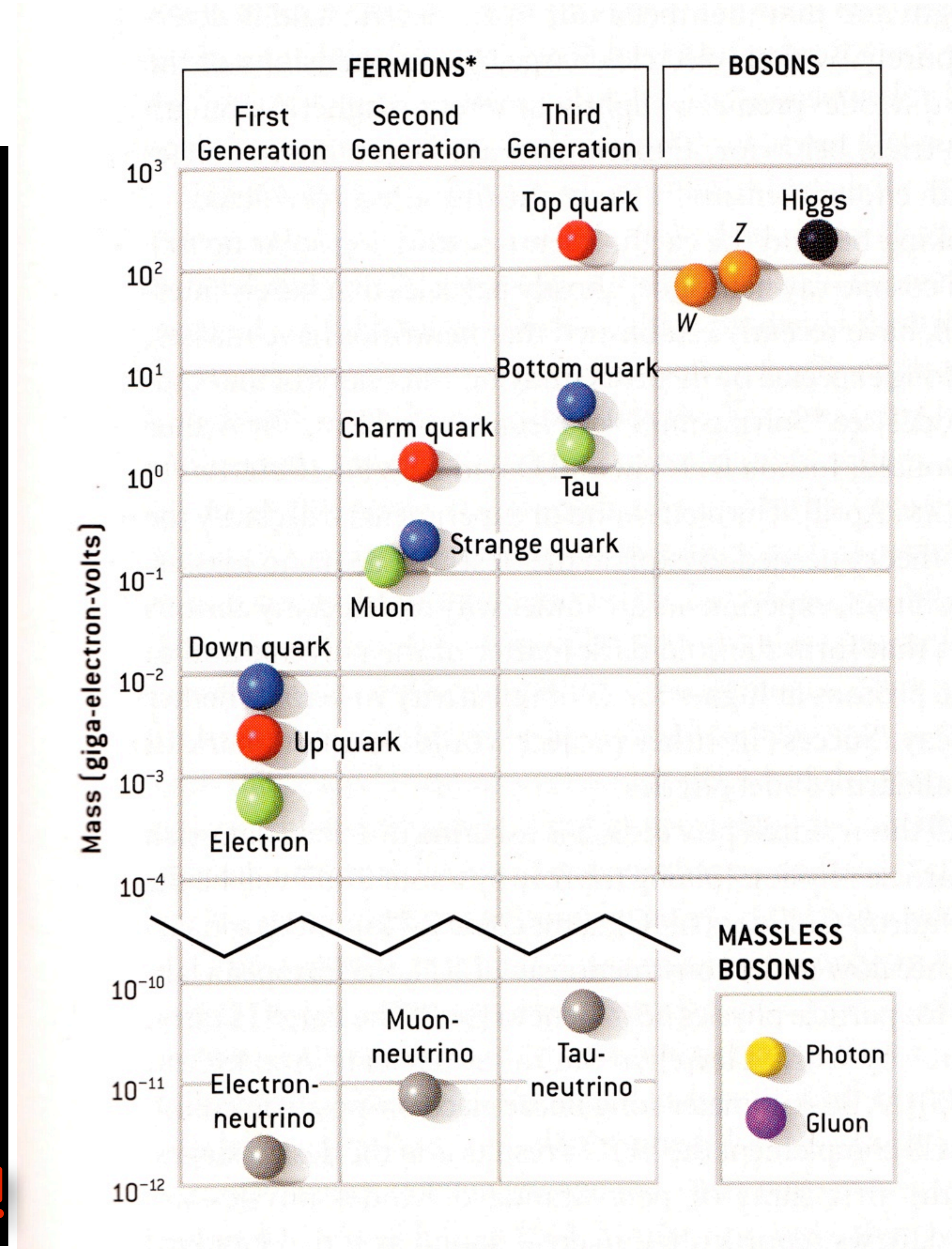
☛

Physics Beyond the Standard Model

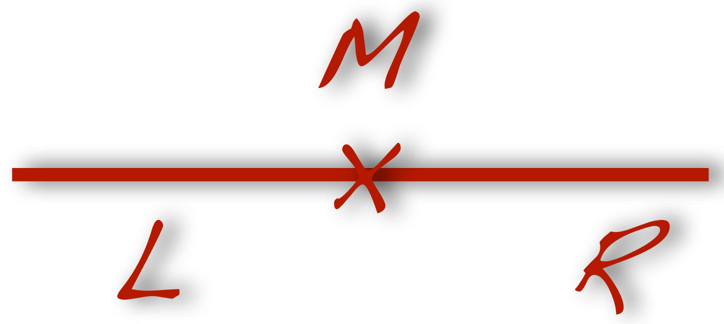
ν Mass Problem



Elementary Particle Masses Span $\geq$ 11 orders of magnitude !



ν Mass Term



① "Poor man's" extension of the SM

If $\nu \neq \nu^{\mathbf{c}} = \mathbf{C}\,\bar{\nu}^{\mathbf{T}}$ Dirac Particle

symmetrize the model, offers no explanation to the smallness of m_ν

$L_\alpha \equiv (2,-1/2)$ $E_\alpha \equiv (1,-1)$ $N_\alpha \equiv (1,-0)$

$$-\mathcal{L}_{\mathrm{Y}} = \mathbf{y}^{\mathbf{d}}_{\alpha\beta}\,\overline{\mathbf{Q}}_\alpha\,\mathbf{\Phi}\,\mathbf{D}_\beta + \mathbf{y}^{\mathbf{u}}_{\alpha\beta}\,\overline{\mathbf{Q}}_\alpha\,\tilde{\mathbf{\Phi}}\,\mathbf{U}_\beta + \mathbf{y}^{\ell}_{\alpha\beta}\overline{\mathbf{L}}_\alpha\,\mathbf{\Phi}\,\mathbf{E}_\beta + \mathrm{h.c.}$$

$$+\,\mathbf{y}^{\nu}_{\alpha\beta}\,\overline{\mathbf{L}}_\alpha\,\tilde{\mathbf{\Phi}}\,\mathbf{N}_\beta + \mathrm{h.c.}$$

EWSB Dirac Mass Term

Higgs acquires a vev ⇨ $-\,\mathbf{m_D}\,\bar{\nu}_{\mathbf{L}}\,\nu_{\mathbf{R}} + \mathrm{h.c.}$

① "Poor man's" extension of the SM

$$-\mathcal{L}_{\rm Y} = \left(\frac{\mathbf{v}+\mathbf{h}}{\sqrt{\mathbf{2}}}\right)\left[\overline{\ell'_{\mathbf{L}}}\,\mathbf{y}^{\ell'}\;\ell'_{\mathbf{R}} + \overline{\mathbf{N}'_{\mathbf{L}}}\,\mathbf{y}^{\nu'}\;\mathbf{N}'_{\mathbf{R}}\right] + \text{h.c.}$$

$$\ell'_{\mathbf{L,R}} \equiv \begin{pmatrix} e' \\ \mu' \\ \tau' \end{pmatrix}_{\mathbf{L,R}}$$

$$\ell_{\mathbf{L,R}} = \mathbf{V}^{\ell\dagger}_{\mathbf{L,R}}\,\ell'_{\mathbf{L,R}}$$

real positive numbers

$$\mathbf{y}^{\ell} = \mathbf{V}^{\ell\dagger}_{\mathbf{L}}\,\mathbf{y}^{\ell'}\;\mathbf{V}^{\ell}_{\mathbf{R}} \qquad \mathbf{y}^{\ell}_{\alpha\beta} = \mathbf{y}^{\ell}_{\alpha}\,\delta_{\alpha\beta}$$

unitary matrices

$$\mathbf{N}'_{\mathbf{L,R}} \equiv \begin{pmatrix} \nu'_e \\ \nu'_\mu \\ \nu'_\tau \end{pmatrix}_{\mathbf{L,R}}$$

$$\mathbf{N}_{\mathbf{L,R}} = \mathbf{V}^{\nu\dagger}_{\mathbf{L,R}}\,\mathbf{N}'_{\mathbf{L,R}}$$

real positive numbers

$$\mathbf{y}^{\nu} = \mathbf{V}^{\nu\dagger}_{\mathbf{L}}\,\mathbf{y}^{\nu'}\;\mathbf{V}^{\nu}_{\mathbf{R}} \qquad \mathbf{y}^{\nu}_{\alpha\beta} = \mathbf{y}^{\nu}_{\alpha}\,\delta_{\alpha\beta}$$

unitary matrices

① "Poor man's" extension of the SM

$$- \mathcal{L}^{\mathbf{D}}_{\text{mass}} = \underbrace{\frac{\mathbf{v}}{\sqrt{\mathbf{2}}}\, \mathbf{y}^{\ell}_{\alpha}}_{\mathbf{m}_{\mathbf{e}_\alpha}} \overline{\mathbf{e}_{\alpha\mathbf{L}}}\, \mathbf{e}_{\alpha\mathbf{R}} + \underbrace{\frac{\mathbf{v}}{\sqrt{\mathbf{2}}}\, \mathbf{y}^{\nu}_{\mathbf{i}}}_{\mathbf{m}_{\nu_\mathbf{i}}} \overline{\nu_{\mathbf{iL}}}\, \nu_{\mathbf{iR}} + \text{h.c.}$$

charged fermions masses

neutrino masses

$$\ell_{\mathbf{L,R}} \equiv \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\mathbf{L,R}} \equiv \begin{pmatrix} e_e \\ e_\mu \\ e_\tau \end{pmatrix}_{\mathbf{L,R}} \qquad \text{new fields} \qquad \mathbf{N}_{\mathbf{L,R}} \equiv \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\mathbf{L,R}}$$

Yukawas have to be fine-tuned to explain smallness of neutrino masses

Ok. But what happens to the CC and NC ?

① "Poor man's" extension of the SM

charged current for leptons

$$\mathbf{j}^{\mu}_{\mathbf{W,L}} = \mathbf{2}\,\overline{\nu}'_{\alpha}\gamma^{\mu}\,\mathbf{P_L}\,\mathbf{e}'_{\alpha} = \mathbf{2}\,\overline{\nu}'_{\alpha\,\mathbf{L}}\,\gamma^{\mu}\,\mathbf{e}'_{\alpha\,\mathbf{L}} = \mathbf{2}\,\overline{\mathbf{N}}'_{\mathbf{L}}\gamma^{\mu}\ell'_{\mathbf{L}}$$

chiral flavor diagonal interaction

$$= \mathbf{2}\,\overline{\mathbf{N}_{\mathbf{L}}}\,\boxed{\mathbf{V}^{\nu\dagger}_{\mathbf{L}}\mathbf{V}^{\ell}_{\mathbf{L}}}\,\gamma^{\mu}\,\ell_{\mathbf{L}} = \mathbf{2}\,\overline{\nu_{\mathbf{iL}}}\,\boxed{\mathbf{U}^{*}_{\alpha\mathbf{i}}}\,\gamma^{\mu}\,\mathbf{e}_{\alpha\mathbf{L}}$$

Mixing Matrix (Pontecorvo, Maki, Sakata, Nakagawa)

define LH flavor neutrinos as $$\nu_{\alpha\mathbf{L}} = \mathbf{U}_{\alpha\mathbf{i}}\,\nu_{\mathbf{iL}}$$

Mixing ⇒ family Lepton Number (L_e, L_μ, L_τ) violated

but Total Lepton Number (L) conserved

① "Poor man's" extension of the SM

neutral current for neutrinos

$$\mathbf{j}^{\mu}_{\mathbf{Z},\nu} = \overline{\nu}_{\alpha \mathbf{L}}\, \gamma^{\mu}\, \nu_{\alpha \mathbf{L}}$$

chiral flavor diagonal interaction

$$= \overline{\nu}_{\mathbf{iL}}\, \gamma^{\mu}\, \nu_{\mathbf{iL}}$$

No Mixing here !

NC is the same (GIM Mechanism)

[S.L.Glashow, J. Iliopolos, L. Maiani, Phys. Rev. D 2, 1285 (1970)]

ν_R is sterile !

L_α Violating Processes

Dirac mass term allows for $\not{L}_e, \not{L}_\mu, \not{L}_\tau$

processes such as: $\mu^\pm \rightarrow e^\pm \gamma$ or $\mu^\pm \rightarrow e^\pm e^+ e^-$

eg. $\mu^\pm \rightarrow e^\pm \gamma$

$$\sum_j U^*_{\mu j} U_{ej} = 0$$

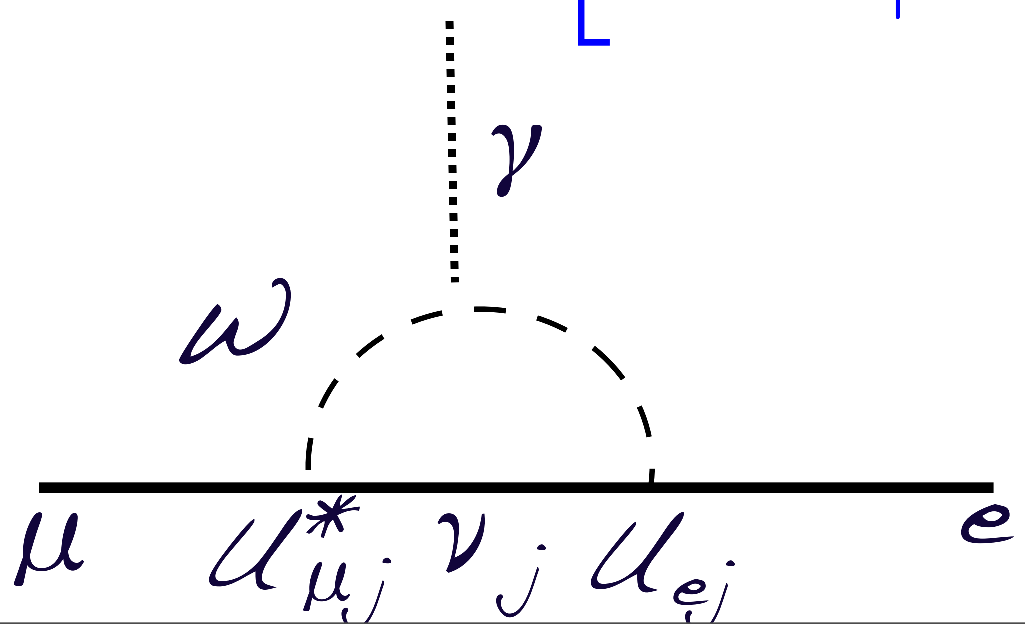
GIM Mechanism

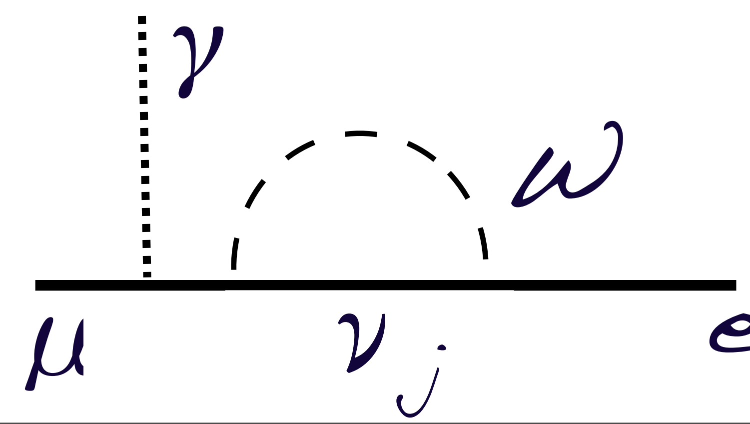
$$\Gamma = \frac{G_F m_\mu^5}{192\pi^3} \left[\frac{3\alpha_{em}}{32\pi} \left| \sum_j U^*_{\mu j} U_{ej} \frac{m_{\nu_i}}{M_W} \right|^2 \right]$$

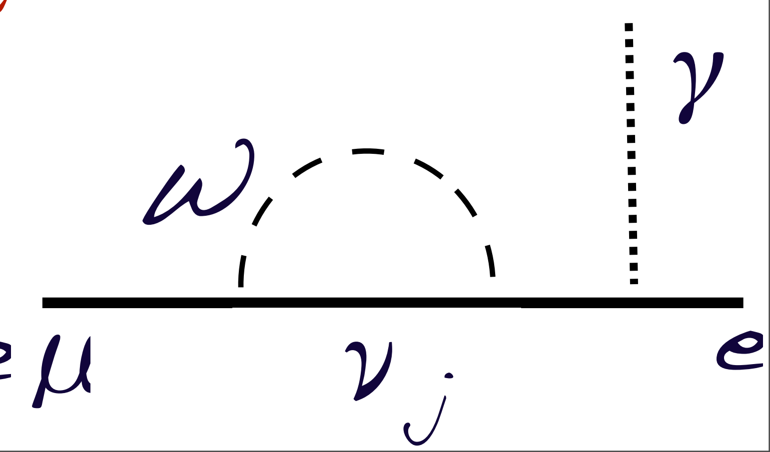
SM : $BR \leq 10^{-25}$

$BR_{exp} \leq 2.4 \times 10^{-12}$

suppression







Phases of U

$$\mathbf{j}^{\mu}_{\mathbf{W,L}} = \mathbf{2}\ \overline{\nu_{\mathbf{iL}}}\ \mathbf{U}^{*}_{\alpha\mathbf{i}}\ \gamma^{\mu}\ \mathbf{e}_{\alpha\mathbf{L}}$$

Can re-phase $\mathbf{e}_{\alpha\mathbf{L}} \longrightarrow \mathbf{e}^{\mathbf{i}\phi_{\alpha}}\ \mathbf{e}_{\alpha\mathbf{L}}$ $\qquad \nu_{\mathbf{iL}} \longrightarrow \mathbf{e}^{\mathbf{i}\phi_{\mathbf{i}}}\ \nu_{\mathbf{iL}}$

$$\mathbf{j}^{\mu}_{\mathbf{W,L}} = \mathbf{2}\ \overline{\nu_{\mathbf{iL}}}\ \mathbf{e}^{-\mathbf{i}(\phi_{\mathbf{1}}-\phi_{\mathbf{e}})}\ \mathbf{e}^{-\mathbf{i}(\phi_{\mathbf{i}}-\phi_{\mathbf{1}})}\ \mathbf{e}^{\mathbf{i}(\phi_{\alpha}-\phi_{\mathbf{e}})}\ \mathbf{U}^{*}_{\alpha\mathbf{i}}\ \gamma^{\mu}\ \mathbf{e}_{\alpha\mathbf{L}}$$

1 $\qquad$ N - 1 $\qquad$ N - 1

$1 + 2(N-1) = 2N-1$ phases can be arbitrarily chosen

N=3 → 5 phases can be eliminated from U

only 1 physical phase

Basic Points :

- we need to introduce singlet R neutrino fields (ν_R)
- we make use of the SM Higgs Mechanism
- $$\mathcal{L}^{\mathrm{D}}_{\mathrm{mass}} = -m\bar{\nu}\,\nu = -m(\bar{\nu}_R\,\nu_L + \bar{\nu}_L\,\nu_R)$$
- mass hierarchy problem remains $$m_j^\nu = \frac{y_j^\nu\, v}{\sqrt{2}}$$
- L_e, L_μ, L_τ are violated
- L is conserved (exact global symmetry at the classical level, just like B)
- generates a mixing matrix analogous to V_{CKM}

② +Clever extensions of the SM

If $\nu = \nu^{\mathbf{c}} = \mathbf{C}\,\bar{\nu}^{\mathbf{T}}$ Majorana Particle

if we introduce ν_R we can have

Majorana Mass Term

$$-\frac{1}{2}\,\mathrm{m_R}\,\overline{\nu_{\mathbf{R}}^{\mathbf{c}}}\,\nu_{\mathbf{R}} + \mathrm{h.c.}$$

L is violated by 2 units

$\mathbf{P_L}\,\nu_{\mathbf{R}}^{\mathbf{c}} = \nu_{\mathbf{R}}^{\mathbf{c}}$ this is invariant under $SU(2)_L \times U(1)_Y$

② +Clever extensions of the SM

If $\nu = \nu^c = C\,\bar{\nu}^T$ Majorana Particle

if we don't introduce ν_R we can have

Majorana Mass Term

$$-\frac{1}{2} m_L \overline{\nu_L^c}\, \nu_L + h.c.$$

L is violated by 2 units

$P_R \nu_L^c = \nu_L^c$

but not invariant under $SU(2)_L \times U(1)_Y$

need to extend the SM ...

Majorana Mass Term

we can write a Majorana mass term with only ν_L

(or ν_R)

$$\mathbf{P_R}\,\nu_L^c = \nu_L^c$$

$$\nu^c = \nu \Longrightarrow \nu = \nu_L + \nu_L^c \Longrightarrow \mathcal{L}^{ML}_{mass} = -\frac{1}{2}\,\mathbf{m_L}\,\overline{\nu_L^c}\,\nu_L + \mathrm{h.c.}$$

the 1/2 factor avoids double counting since ν_L and ν_L^c are not independent

$$\mathcal{L}^{ML} = \frac{1}{2}\left[\overline{\nu}_L\,\imath\not\partial\nu_L + \overline{\nu}_L^c\,\imath\not\partial\nu_L^c - \mathbf{m_L}\left(\overline{\nu}_L^c\,\nu_L + \overline{\nu}_L\nu_L^c\right)\right]$$

$$\mathcal{L}^{ML}_{mass} = \frac{\mathbf{m_L}}{2}\left(\nu_L^T\,\mathbf{C}^\dagger\,\nu_L + \nu_L^\dagger\,\mathbf{C}\,\nu_L^*\right)$$

Basic Points:

- ☛ no need to introduce singlet R fields (ν_R)
- ☛ use $\nu_{\mathbf{R}} \rightarrow \nu_{\mathbf{L}}^{\mathbf{c}} = \mathbf{C}\,\overline{\nu_{\mathbf{L}}}^{\mathbf{T}}$ and $\nu = \nu^{\mathbf{c}}$
- ☛ $\nu = \nu_{\mathbf{L}} + \nu_{\mathbf{R}} = \nu_{\mathbf{L}} + \mathbf{C}\,\overline{\nu_{\mathbf{L}}}^{\mathbf{T}}$

$$\mathcal{L}_{\mathrm{mass}}^{\mathrm{ML}} = -\frac{\mathbf{m}}{\mathbf{2}}(\overline{\nu_{\mathbf{L}}^{\mathbf{c}}}\,\nu_{\mathbf{L}} + \mathrm{h.c.})$$

- ☛ need a Higgs triplet ($Y=1$) to form a $SU(2)_L \otimes U(1)_Y$ invariant term ($L\Delta L$)
- ☛ L_e, L_μ, L_τ are violated
- ☛ L is also violated by 2 units

The most general mass term is a Dirac-Majorana Mass Term

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \mathcal{L}_{\text{mass}}^{\text{D}} + \mathcal{L}_{\text{mass}}^{\text{ML}} + \mathcal{L}_{\text{mass}}^{\text{MR}}$$

$$\mathcal{L}_{\text{mass}}^{\text{D}} = -\mathbf{m_D}\,\overline{\nu}_{\mathbf{R}}\,\nu_{\mathbf{L}} + \text{h.c.}$$

Dirac Mass Term

$$\mathcal{L}_{\text{mass}}^{\text{ML}} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{m_L}\,\nu_{\mathbf{L}}^{\mathbf{T}}\,\mathbf{C}^{\dagger}\,\nu_{\mathbf{L}} + \text{h.c.}$$

Majorana Mass Term

$$\mathcal{L}_{\text{mass}}^{\text{MR}} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{m_R}\,\nu_{\mathbf{R}}^{\mathbf{T}}\,\mathbf{C}^{\dagger}\,\nu_{\mathbf{R}} + \text{h.c.}$$

Majorana Mass Term

Mixing in General

$$\mathbf{N}'_{\mathbf{L}} \equiv \begin{pmatrix} \nu'_L \\ \nu'^c_R \end{pmatrix} \qquad \nu'_{\mathbf{L}} \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \nu'^{\mathbf{c}}_{\mathbf{R}} \equiv \begin{pmatrix} \nu'^c_{1R} \\ \cdot \\ \cdot \\ \nu'^c_{N_s R} \end{pmatrix}$$

$$\mathcal{L}^{\mathrm{D+M}}_{\mathrm{mass}} \equiv \frac{\mathbf{1}}{\mathbf{2}} \mathbf{N}'^{\mathbf{T}}_{\mathbf{L}} \mathbf{C}^{\dagger} \mathbf{M}^{\mathbf{D+M}} \mathbf{N}'_{\mathbf{L}} + \mathrm{h.c.} \quad \mathbf{M}^{\mathbf{D+M}} = \begin{pmatrix} \cancel{M^L} & (M^D)^T \\ M^D & M^R \end{pmatrix}$$

- Diagonalization of the Dirac-Majorana Mass Term ⇒ Massive Majorana Neutrinos

seesaw formula: $(M_i \gg m_{D\,i}) \Rightarrow$ $m_\nu = -m_D \frac{1}{M_R} m_D^T$

m_D, m_ν and M^R are complex matrices ⇒ natural source of CP violation

➪ Leptogenesis

- 3 new heavy RH neutrinos N_1,N_2,N_3 with masses $M_{ew} \ll M_1 \leq M_2 \leq M_3$

next week ...

Dirac-Majorana

$$\mathbf{M^{D+M}} = \begin{pmatrix} 0 & (M^D)^T \\ M^D & M^R \end{pmatrix}$$

complex symmetric matrix

M_D is a 3xm complex matrix M_R is a mxm symmetric matrix

(a) mass eigenvalues of $M^R \gg v \Rightarrow$ framework of seesaw mechanism sterile neutrinos integrated out & get a low energy effective theory with 3 light active Majorana neutrinos

(b) some mass eigenvalues of $M^R \leq v \Rightarrow$ more than 3 light Majorana neutrinos

(c) $M^R = 0 \Rightarrow$ equivalent to impose L conservation, m=3 and we can identify the 3 sterile neutrinos c/ RH components of the LH fields (Dirac Neutrinos)

Neutrino Mass

&

The Standard Seesaw Mechanisms

Effective Lagrangian Perspective

SM is an Effective Lower Energy Theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \delta\mathcal{L}^{\bf d=5} + \delta\mathcal{L}^{\bf d=6} + \ldots$$

non-renormalizable higher-dimension operators invariant under $SU(2)_L \times U(1)_Y$ made of SM fields active @ low energies with coefficients (model dependent) weighted by inverse powers of Λ (new physics scale)

$$\delta\mathcal{L}^{\bf d=5} = \frac{\bf g}{\bf \Lambda}\left(\mathbf{L}^{\bf T}\sigma_{\bf 2}\mathbf{\Phi}\right)\mathbf{C}^{\dagger}\left(\mathbf{\Phi}^{\bf T}\sigma_{\bf 2}\mathbf{L}\right) + \text{h.c.}$$

[S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979)]

only possible d=5 operator

$$\delta\mathcal{L}^{\bf d=5} \xRightarrow{\text{EWSB}} \delta\mathcal{L}^{\bf M}_{\rm mass} = \frac{\bf 1}{\bf 2}\frac{\bf gv^2}{\bf \Lambda}\nu_{\bf L}^{\bf T}\,\mathbf{C}^{\dagger}\,\nu_{\bf L} + \text{h.c.}$$

Majorana Mass

Tree-level Realizations

[E. Ma, Phys. Rev. Lett. 81, 1171 (1998)]

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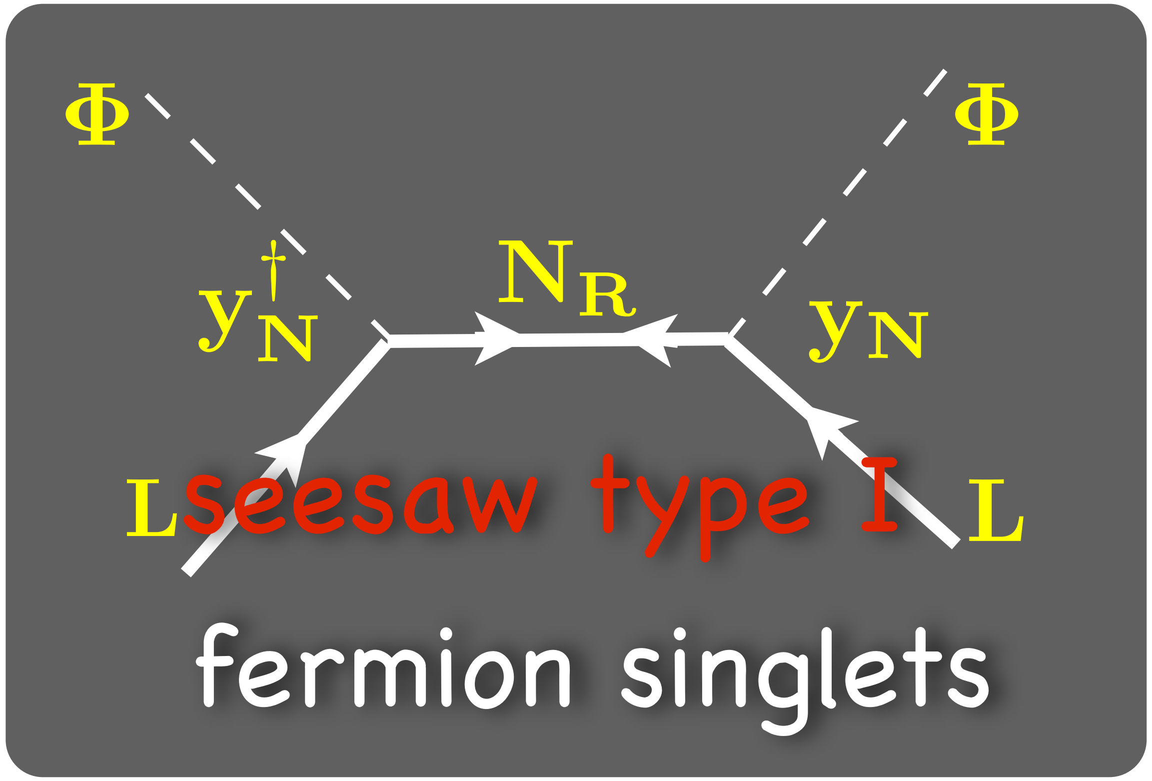

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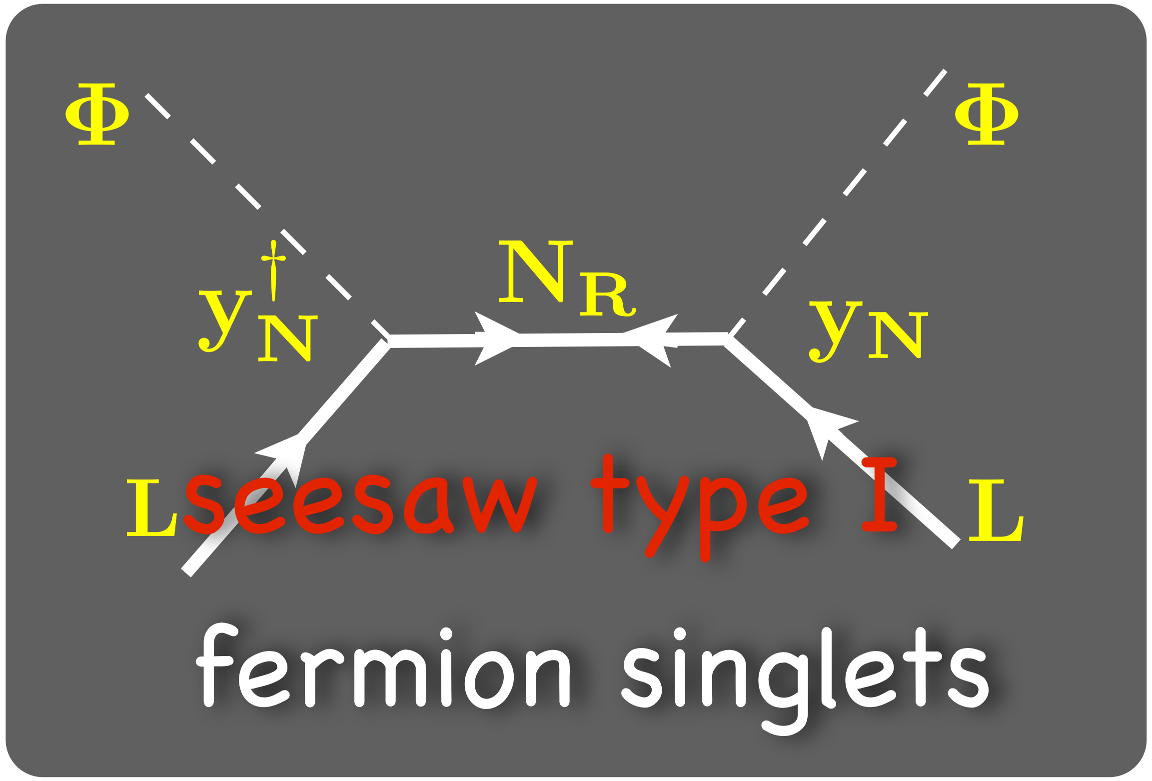

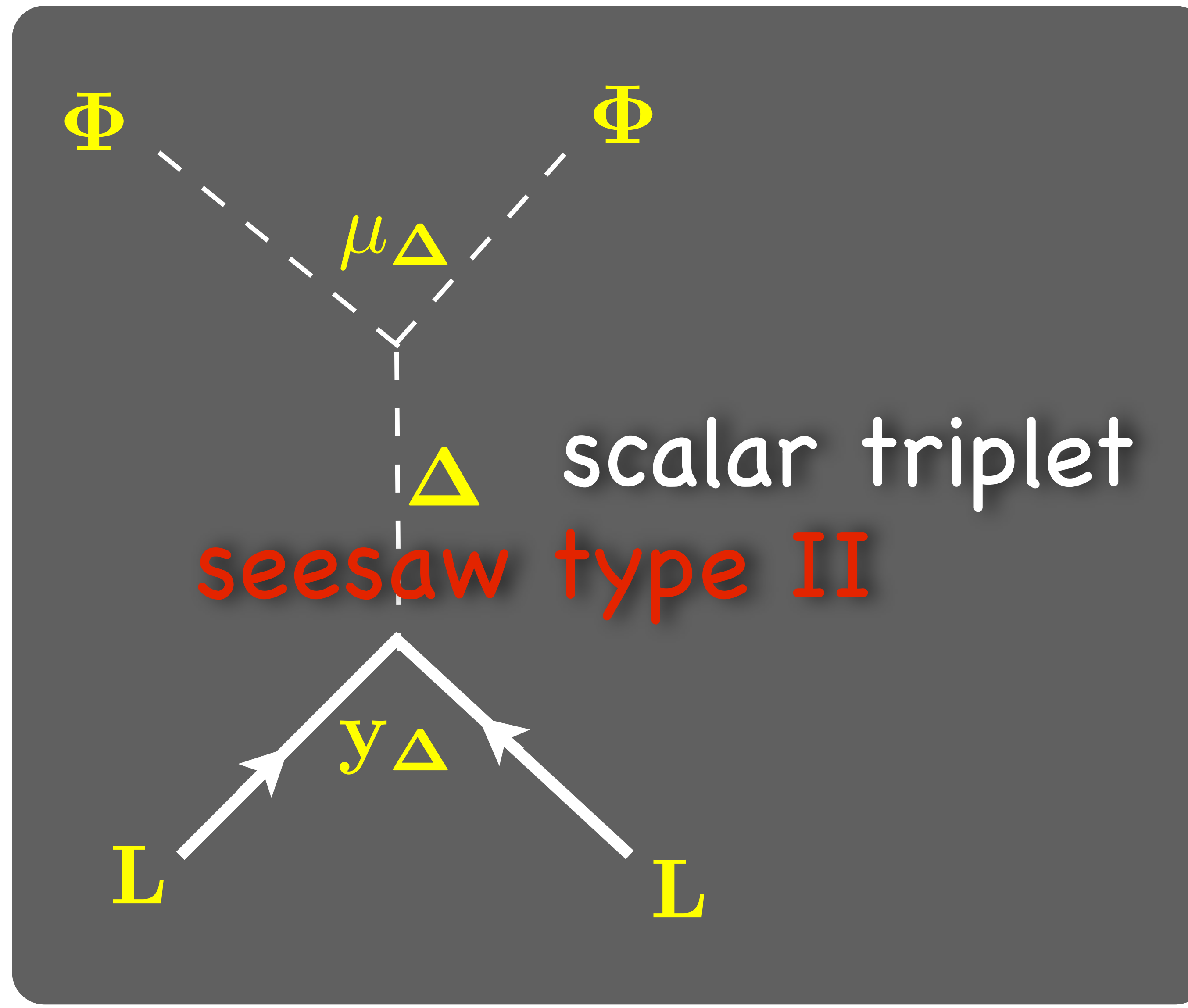

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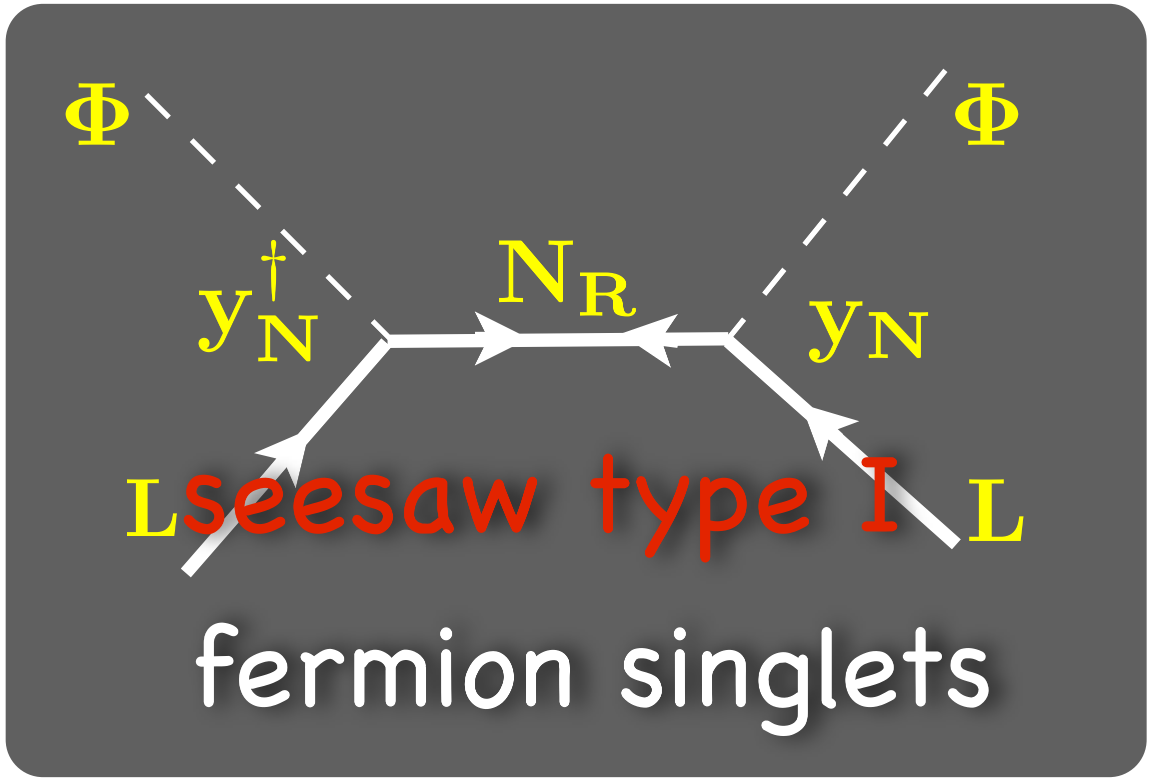
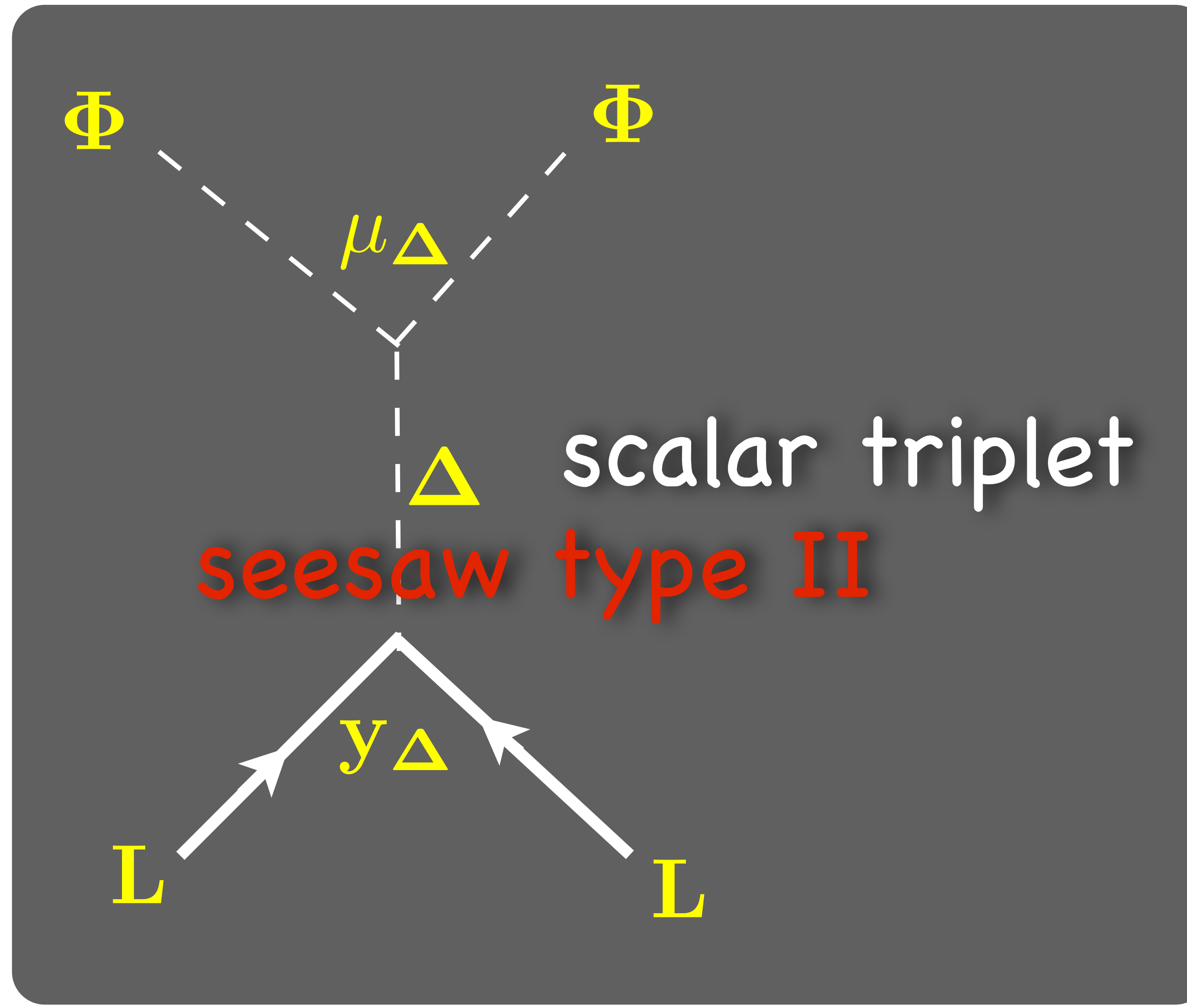
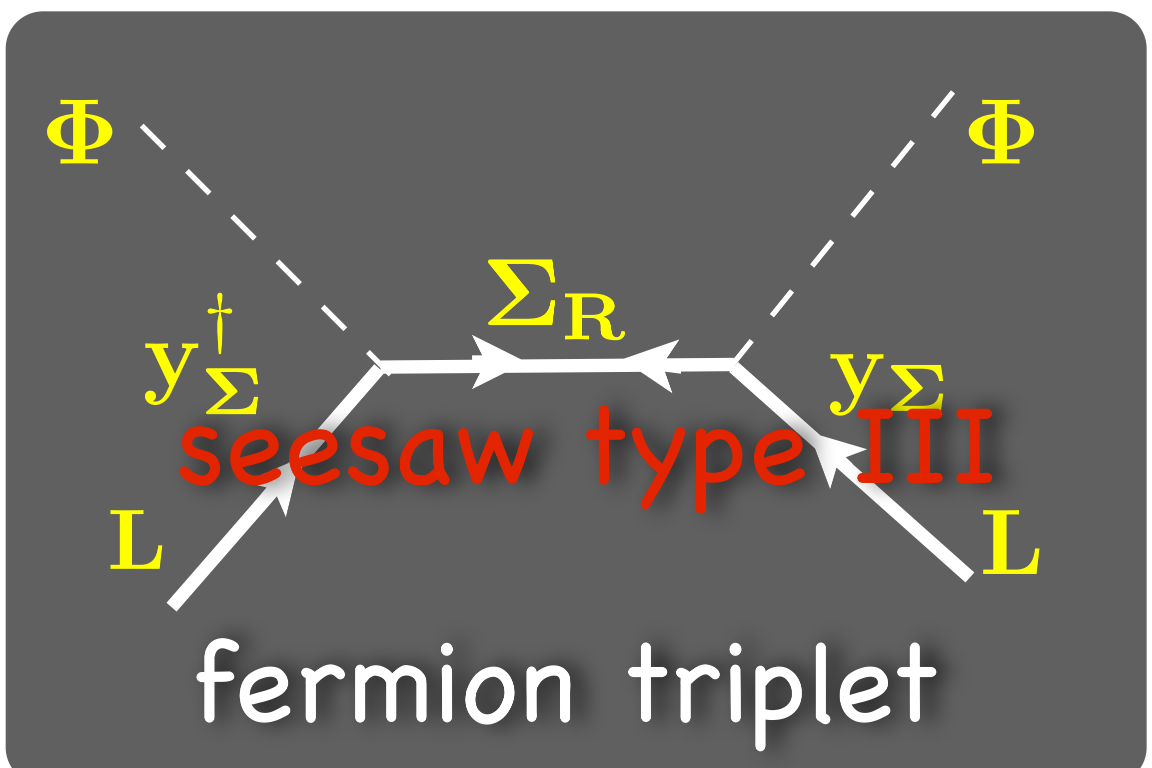






Effective Lagrangian Perspective

integrate out the heavy fields

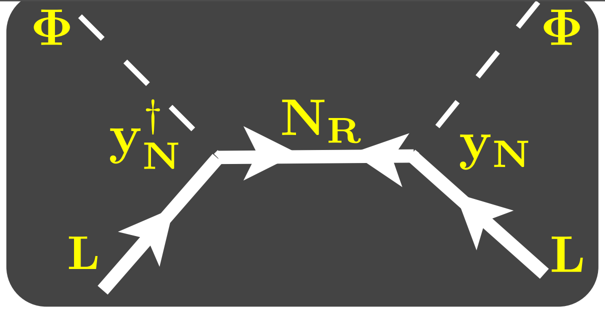
$$\mathbf{e^{iS_{eff}}} = \exp\left\{\mathbf{i}\int \mathbf{d^4x}\,\mathcal{L}_{\mathbf{eff}}(\mathbf{x})\right\}$$

$$\equiv \int \mathcal{D}\mathbf{N}\,\mathcal{D}\overline{\mathbf{N}}\,\mathbf{e^{iS}} = \mathbf{e^{iS_{SM}}}\int \mathcal{D}\mathbf{N}\,\mathcal{D}\overline{\mathbf{N}}\,\mathbf{e^{iS_N}}$$

effective Lagrangian obtained by functional integration over heavy fields

$S_N[N_0]$: N_0 is the solution of the classical equations of motion for the N-field

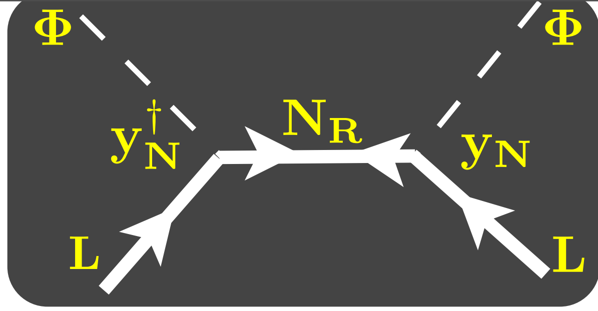
$$\frac{1}{\not{D} - M} = -\frac{1}{M} + \frac{1}{M}\not{D}\frac{1}{M} + \ldots$$ fermions



Seesaw Type I

[P. Minkowski, Phys. Lett. B 67, 421 (1977); M. Gell-Mann, P. Ramond and R. Slansky, Supergravity (1979); T. Yanagida, Proc. Workshop on the Unif. Theo. and the Baryon Numb. in the Univ. (1979); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980)]

minimal seesaw Lagrangian: only add R neutrinos to SM



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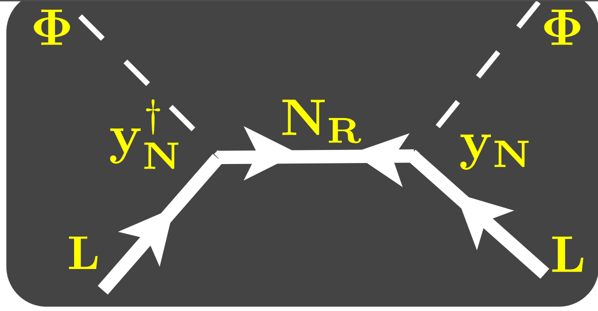
$$\mathcal{L}_{\rm KE} = \imath\,\overline{\mathbf{L}}\,\not{\mathbf{D}}\,\mathbf{L} + \imath\,\overline{\mathbf{R}}\,\not{\mathbf{D}}\,\mathbf{R} + \imath\,\overline{\mathbf{N}}_{\mathbf{R}}\,\not{\partial}\,\mathbf{N}_{\mathbf{R}}$$

SM lepton doublets — SM lepton singlets — R neutrino singlets

New Physics Scale

$$\mathcal{L}_{\rm Y} = -\overline{\mathbf{L}}\,\mathbf{\Phi}\,\mathbf{y}_\ell\,\mathbf{R} - \overline{\mathbf{L}}\,\tilde{\mathbf{\Phi}}\,\mathbf{y}_{\mathbf{N}}^\dagger\,\mathbf{N}_{\mathbf{R}} - \frac{1}{2}\overline{\mathbf{N}}_{\mathbf{R}}\,\mathbf{M}_{\mathbf{R}}\,\mathbf{N}_{\mathbf{R}}^{\mathbf{c}} + \mathrm{h.c.}$$



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minimal seesaw Lagrangian: only add R neutrinos to SM

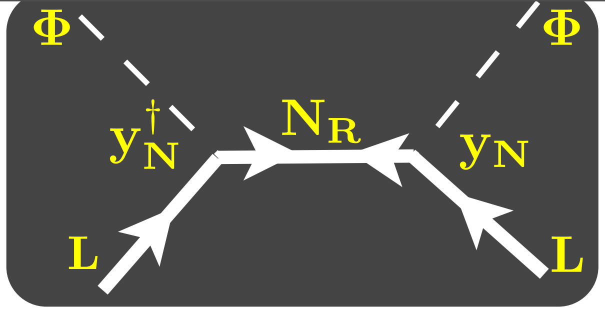
$$\mathcal{L}_{KE} = \imath \, \overline{\mathbf{L}} \, \not{\mathbf{D}} \, \mathbf{L} + \imath \, \overline{\mathbf{R}} \, \not{\mathbf{D}} \, \mathbf{R} + \imath \, \overline{\mathbf{N}}_{\mathbf{R}} \not{\partial} \mathbf{N}_{\mathbf{R}}$$

SM lepton doublets — SM lepton singlets — R neutrino singlets

New Physics Scale

$$\mathcal{L}_{Y} = -\overline{\mathbf{L}} \, \mathbf{\Phi} \, \mathbf{y}_\ell \, \mathbf{R} - \overline{\mathbf{L}} \, \tilde{\mathbf{\Phi}} \, \mathbf{y}_{\mathbf{N}}^\dagger \, \mathbf{N}_{\mathbf{R}} - \frac{1}{2} \overline{\mathbf{N}}_{\mathbf{R}} \, \mathbf{M}_{\mathbf{R}} \, \mathbf{N}_{\mathbf{R}}^{\mathbf{c}} + \mathrm{h.c.}$$

$$\mathbf{m}_\nu \equiv \frac{\mathbf{g}}{\mathbf{\Lambda}} \mathbf{v}^2 = -\frac{1}{2} \mathbf{y}_{\mathbf{N}}^{\mathbf{T}} \frac{1}{\mathbf{M}_{\mathbf{R}}} \mathbf{y}_{\mathbf{N}} \, \mathbf{v}^2$$



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SM lepton doublets — SM lepton singlets — R neutrino singlets

New Physics Scale

$$\mathcal{L}_Y = -\overline{\mathbf{L}}\, \mathbf{\Phi}\, \mathbf{y}_\ell\, \mathbf{R} - \overline{\mathbf{L}}\, \tilde{\mathbf{\Phi}}\, \mathbf{y}_\mathbf{N}^\dagger\, \mathbf{N}_\mathbf{R} - \frac{1}{2} \overline{\mathbf{N}}_\mathbf{R}\, \mathbf{M}_\mathbf{R}\, \mathbf{N}_\mathbf{R}^\mathbf{c} + \mathrm{h.c.}$$

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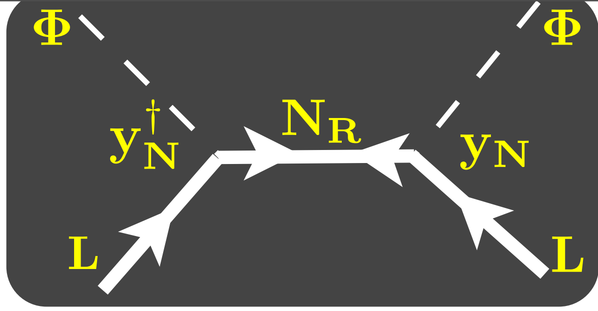
M_R should of the order 10^{11} TeV (10^5 TeV) for $y_N \approx 1$ ($y_N \approx 10^{-3}$)



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SM lepton doublets — SM lepton singlets — R neutrino singlets

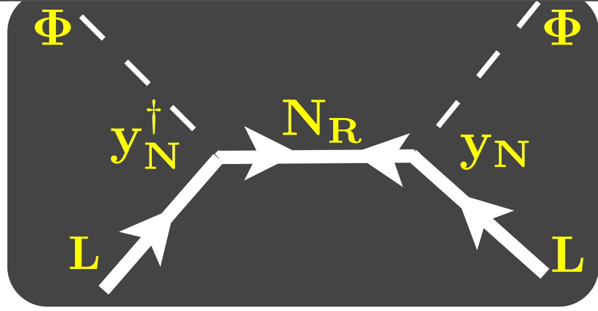
New Physics S…

$$\mathcal{L}_{\rm Y} = -\overline{\mathbf{L}}\,\mathbf{\Phi}\,\mathbf{y}_\ell\,\mathbf{R} - \overline{\mathbf{L}}\,\tilde{\mathbf{\Phi}}\,\mathbf{y}_{\mathbf{N}}^\dagger\,\mathbf{N}_{\mathbf{R}} - \frac{1}{2}\,\overline{\mathbf{N}}_{\mathbf{R}}\,\mathbf{M} \ldots$$

$$\mathbf{m}_\nu \equiv \frac{\mathbf{g}}{\ldots} \ldots \frac{\ldots}{\mathbf{M}_{\mathbf{R}}}\,\mathbf{y}_{\mathbf{N}}\,\mathbf{v}^2$$

Needs 3 N_R to give mass to 3 ν

M… d of the order 10^{11} TeV (10^5 TeV) for $y_N \approx 1$ ($y_N \approx 10^{-3}$)



Seesaw Type I

[A. Broncano, M.B. Gavela and E.E. Jenkins, Phys. Lett. B 552, 177 (2003); Nucl. Phys. B 672, 163 (2003)]

only one d=6 tree level operator

$$\delta\mathcal{L}^{d=6} = g^{d=6}\,(\overline{L}\,\tilde{\Phi})\, i\partial\!\!\!/\,(\tilde{\Phi}^\dagger\, L)$$

⇨ EWSB corrections to d=4 KE terms of L leptons

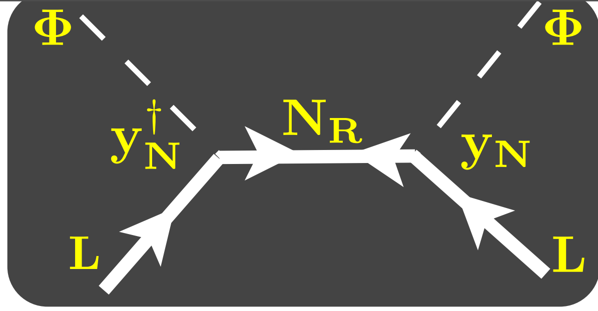
⇩

non-unitary low-energy leptonic mixing matrix

where

$$g^{d=6} = y_N^\dagger \frac{1}{M_R^\dagger}\frac{1}{M_R} y_N$$

quadratically suppressed



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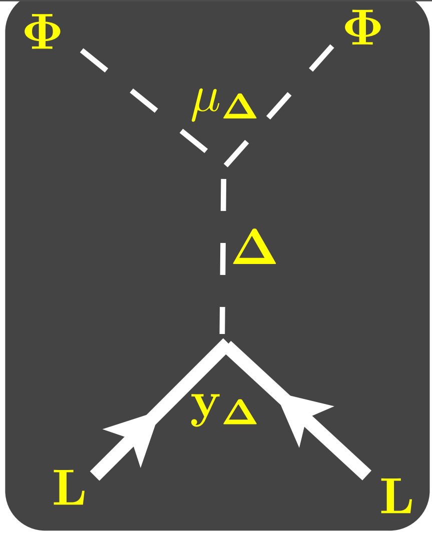
quadratically suppressed

$$U \rightarrow N = \left(1 - \frac{\epsilon}{2}\right) U \qquad NN^\dagger = (1-\epsilon) \qquad N^\dagger N = U^\dagger(1-\epsilon)U$$

with $\epsilon \equiv \frac{v^2}{2} g^{d=6}$ $\qquad j^{\mu}_{Z,\nu} \equiv \frac{1}{2}\overline{\nu}_i \gamma^\mu (N^\dagger N)_{ij}\, \nu_j$



Seesaw Type II

[M. Magg and C. Wetterich, Phys. Lett. B 94, 61 (1980); J. Schechter and J.W.F. Valle, Phys. Rev. D 22, 2227 (1980); R.N. Mohapatra and G. Senjanovic, Phys. Rev. D 23, 165 (1981)]

add $SU(2)_L$ triplet scalar field $\vec{\Delta}$ (Y=1)

minimal Lagrangian gauge invariant allows for

$$\mathcal{L}_{\Delta,(\mathrm{L},\Phi)} = \bar{\tilde{L}}\, y_\Delta\, (\vec{\sigma}\cdot\vec{\Delta})\, L + \mu_\Delta\, \tilde{\Phi}^\dagger (\vec{\sigma}\cdot\vec{\Delta})^\dagger\, \Phi + \mathrm{h.c.}$$

coupling to SM lepton doublets — coupling to SM Higgs doublet

$$\vec{\Delta} = (\Delta_1, \Delta_2, \Delta_3)$$

where

$$\tilde{L} = \imath\, \sigma_2\, (L)^c$$

physical fields

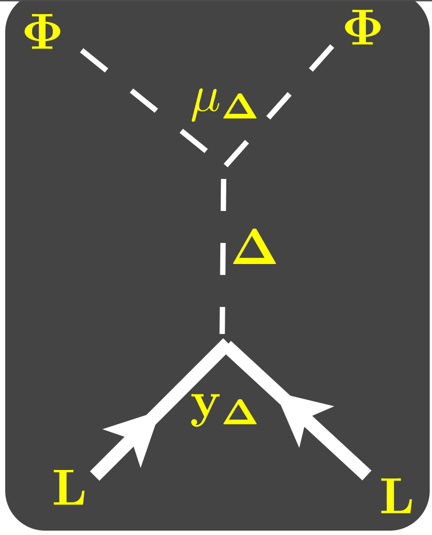
$$\langle \mathbf{\Delta^0} \rangle = \mathbf{u}/\sqrt{\mathbf{2}} = \mu_{\mathbf{\Delta}} \mathbf{v^2}/(\sqrt{\mathbf{2}}\, \mathbf{M^2_\Delta})$$

$$\Delta^{++} \equiv \frac{1}{\sqrt{2}}(\Delta_1 - \imath\Delta_2) \quad \Delta^+ \equiv \Delta_3 \quad \Delta^0 \equiv \frac{1}{\sqrt{2}}(\Delta_1 + \imath\Delta_2)$$

$$\mathbf{m}_\nu \equiv \frac{\mathbf{g}}{\mathbf{\Lambda}}\,\mathbf{v^2} = -\mathbf{2}\,\mathbf{y_\Delta}\,\mathbf{u} = -\mathbf{2}\,\mathbf{y_\Delta}\,\frac{\mu_{\mathbf{\Delta}}}{\mathbf{M^2_\Delta}}\,\mathbf{v^2}$$

New Physics Scale

Majorana Mass Matrix for light neutrinos



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$$\vec{\Delta} = (\Delta_1, \Delta_2, \Delta_3)$$

where

$$\tilde{L} = \imath\sigma_2 (\ldots$$ [illegible]

physical fields

$$\langle \mathbf{\Delta^0} \rangle \ldots / (\sqrt{2}\, \mathbf{M_\Delta^2})$$ [illegible]

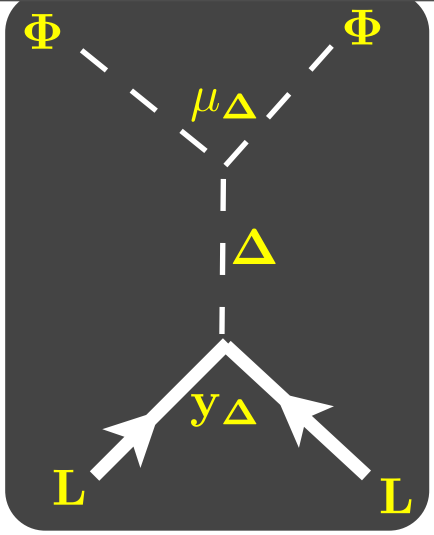
$$\Delta^{++} \equiv \frac{1}{\sqrt{2}}(\Delta_1 \ldots \Delta_3 \qquad \Delta^0 \equiv \frac{1}{\sqrt{2}}(\Delta_1 + \imath\Delta_2)$$ [illegible]

$$\ldots \frac{\ldots}{\mathbf{\Lambda}}\, \mathbf{v^2} = -2\, \mathbf{y_\Delta}\, \mathbf{u} = -2\, \mathbf{y_\Delta}\, \frac{\mu_\Delta}{\mathbf{M_\Delta^2}}\, \mathbf{v^2}$$ [illegible]

New Physics Scale

Majorana Mass Matrix for light neutrinos

One Δ can give mass to 3 ν



Seesaw Type II

[A. Abada, C. Biggio, F. Bonnet, M. B. Gavela and T. Hambye, JHEP 12, 061 (2007)]

three d=6 tree level operators

$$\delta\mathcal{L}_{4\mathrm{L}} = \frac{1}{M_\Delta^2}\left(\overline{\tilde{L}}\, y_\Delta\, \vec{\sigma}\, L\right)\left(\overline{L}\,\vec{\sigma}\, y_\Delta^\dagger\, \tilde{L}\right)$$

quartic ΔΦ couplings

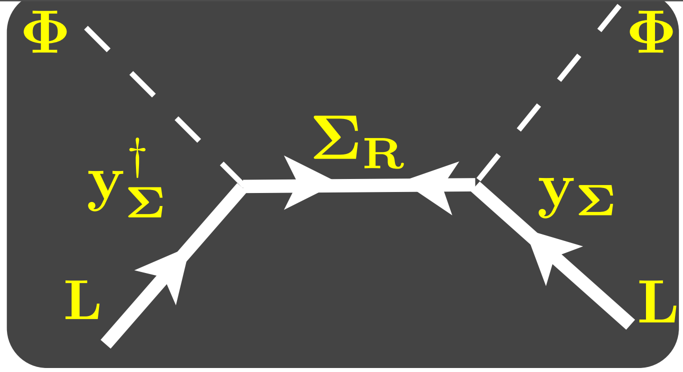
$$\delta\mathcal{L}_{6\Phi} = -2(\lambda_3+\lambda_5)\frac{|\mu_\Delta|^2}{M_\Delta^4}\left(\Phi^\dagger\Phi\right)^3$$

$$\delta\mathcal{L}_{\Phi\mathrm{D}} = \frac{|\mu_\Delta|^2}{M_\Delta^4}\left(\Phi^\dagger\vec{\sigma}\,\tilde{\Phi}\right)\left(\overleftarrow{D}_\mu\vec{D}^\mu\right)\left(\tilde{\Phi}^\dagger\vec{\sigma}\,\Phi\right)$$

many deviations from SM predictions

but no non-unitary mixing matrix !

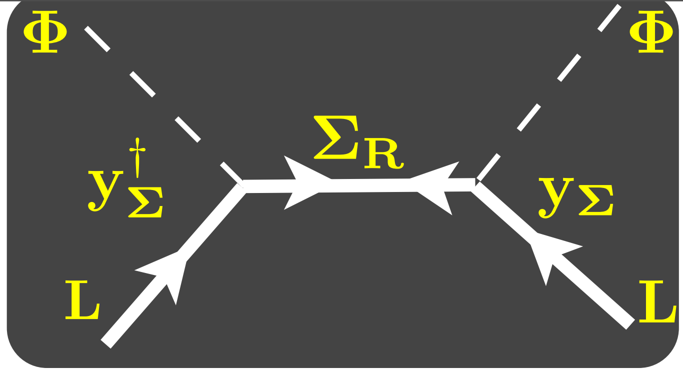
[G_F gets a correction, v gets shifted, M_Z gets a correction etc.]



Seesaw Type III

[R. Foot, H. Lew, X.G.He and G.C. Joshi, Z. Phys. C 44, 441 (1989); E. Ma, Phys. Rev. Lett. 81, 1171 (1998)]

add $SU(2)_L$ fermion triplet $\vec{\Sigma}$ (Y=0)



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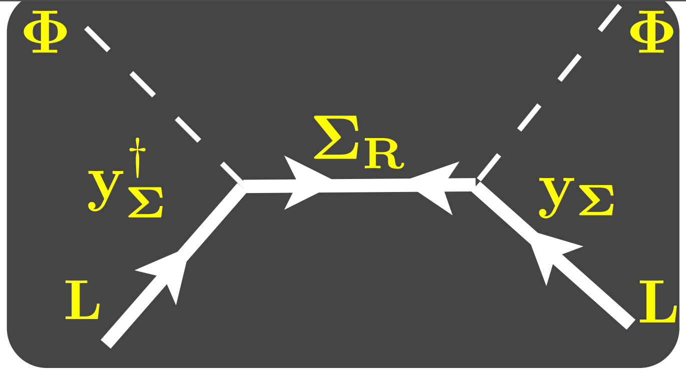
Majorana Mass Term — coupling with L and Φ

$$\vec{\Sigma} = (\Sigma_1, \Sigma_2, \Sigma_3)$$

$$\Sigma^\pm \equiv \frac{1}{\sqrt{2}}(\Sigma_1 \mp \imath\, \Sigma_2) \quad \Sigma^0 \equiv \Sigma_3$$



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$$\Sigma^\pm \equiv \frac{1}{\sqrt{2}} (\Sigma_1 \mp \imath \Sigma_2) \quad \Sigma^0 \equiv \Sigma_3$$

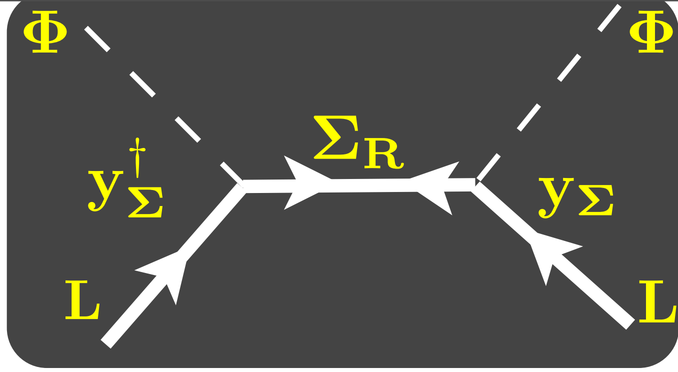
if $M_\Sigma >> v$

EWSB ⇨

$$\mathbf{m}_\nu \equiv \frac{\mathbf{g}}{\mathbf{\Lambda}} \mathbf{v}^2 = -\mathbf{y}_\Sigma^T \frac{1}{2 M_\Sigma} \mathbf{y}_\Sigma \mathbf{v}^2$$

New Physics Scale

Majorana Mass Matrix for light neutrinos



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$$\Sigma^\pm \equiv \frac{1}{\sqrt{2}} (\Sigma_1 \mp \imath \Sigma_2)$$

if $M_\Sigma \gg v$

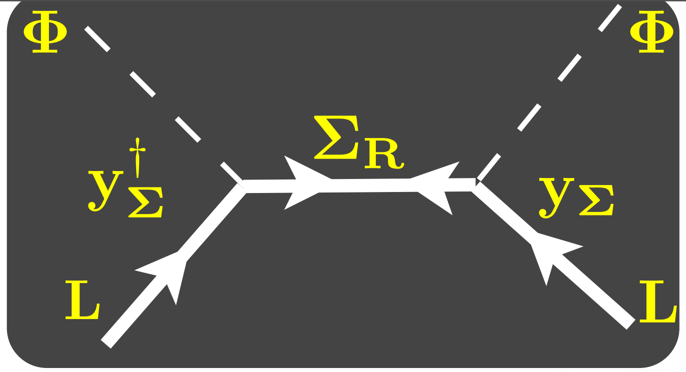
EWSB ⇨

$$\ldots y_\Sigma \frac{1}{2 M_\Sigma} y_\Sigma v^2$$

New Physics Scale

Majorana Mass Matrix for light neutrinos

One Σ can give mass to 3 ν



Seesaw Type III

[A. Abada, C. Biggio, F. Bonnet, M. B. Gavela and T. Hambye, JHEP 12, 061 (2007)]

only one d=6 tree level operator

$$\delta\mathcal{L}^{d=6} = \mathbf{g}^{d=6}\,(\overline{\mathbf{L}}\,\vec{\sigma}\,\tilde{\mathbf{\Phi}})\,i\not{D}\,(\tilde{\mathbf{\Phi}}^\dagger\,\vec{\sigma}\,\mathbf{L})$$

$\Rightarrow$ EWSB corrections to d=4 KE terms of light leptons and their coupling to W

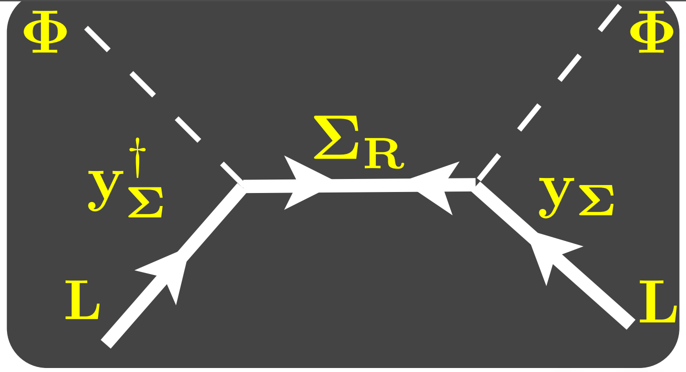
where

$$\mathbf{g}^{d=6} = \mathbf{y}_\Sigma^\dagger\,\frac{1}{\mathbf{M}_\Sigma^\dagger}\,\frac{1}{\mathbf{M}_\Sigma}\,\mathbf{y}_\Sigma$$

quadratically suppressed

$\Downarrow$

non-unitary low-energy leptonic mixing matrix



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quadratically suppressed

$\Downarrow$

non-unitary low-energy leptonic mixing matrix

$$\mathbf{U} \to \mathbf{N} = \left(\mathbf{1} + \frac{\epsilon^{\mathbf{\Sigma}}}{\mathbf{2}}\right)\mathbf{U} \qquad \mathbf{N}\mathbf{N}^\dagger = (\mathbf{1} + \epsilon^{\mathbf{\Sigma}}) \qquad \mathbf{N}^\dagger\mathbf{N} = \mathbf{U}^\dagger(\mathbf{1} + \epsilon^{\mathbf{\Sigma}})\mathbf{U}$$

with $\epsilon^{\mathbf{\Sigma}} \equiv \frac{\mathbf{v}^2}{\mathbf{2}}\,\mathbf{g}^{d=6}$ $\qquad \mathbf{j}_3^\mu(\nu) \equiv \frac{\mathbf{1}}{\mathbf{2}}\overline{\nu}\,\gamma^\mu\,(\mathbf{N}^\dagger\mathbf{N})^{-\mathbf{1}}\,\nu \qquad \mathbf{j}_3^\mu(\ell) \equiv \frac{\mathbf{1}}{\mathbf{2}}\overline{\ell}\,\gamma^\mu\,(\mathbf{N}\mathbf{N}^\dagger)^{\mathbf{2}}\,\ell$

About the Seesaw Scale

Λ = regulator cutoff

one-loop contribution to the Higgs mass

$$\delta m_H^2 = -\frac{y_N^\dagger y_N}{16\pi^2}\left[2\,\Lambda^2 + 2\,M_N^2 \log\frac{M_N^2}{\Lambda^2}\right]$$

[J.A. Casa, J. R. Espinosa and I. Hidalgo, JHEP 11, 057 (2004)]

$$\delta m_H^2 = \frac{1}{16\pi^2}\left[3\,\lambda_3\left(\Lambda^2 - M_\Delta^2 \log\frac{\Lambda^2}{M_\Delta^2}\right) - 12\,|\mu_\Delta|^2 \log\frac{\Lambda^2}{M_\Delta^2}\right]$$

[A. Abada, C. Biggio, F. Bonnet, M. B. Gavela and T. Hambye, JHEP 12, 061 (2007)]

$$\delta m_H^2 = -3\,\frac{y_\Sigma^\dagger y_\Sigma}{16\pi^2}\left[2\,\Lambda^2 + 2\,M_\Sigma^2 \log\frac{M_\Sigma^2}{\Lambda^2}\right]$$

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seesaw type I

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seesaw type III

About the Seesaw Scale

Λ = regulator cutoff

one-loop contribution to the Higgs mass

$$\delta m_H^2 = -\frac{y_N^\dagger y_N}{16\pi^2}\left[2\,\Lambda^2 + 2\,M_N^2 \log\frac{M_N^2}{\Lambda^2}\right]$$

seesaw type I

[J.A. Casa, J. R. Espinosa and I. Hidalgo, JHEP 11, 057 (2004)]

seesaw type II

$$\delta m_H^2 = \frac{1}{16\pi^2}\left[3\,\lambda_3\left(\Lambda^2 - M_\Delta^2 \log\frac{\Lambda^2}{M_\Delta^2}\right) - 12\,|\mu_\Delta|^2 \log\frac{\Lambda^2}{M_\Delta^2}\right]$$

[A. Abada, C. Biggio, F. Bonnet, M. B. Gavela and T. Hambye, JHEP 12, 061 (2007)]

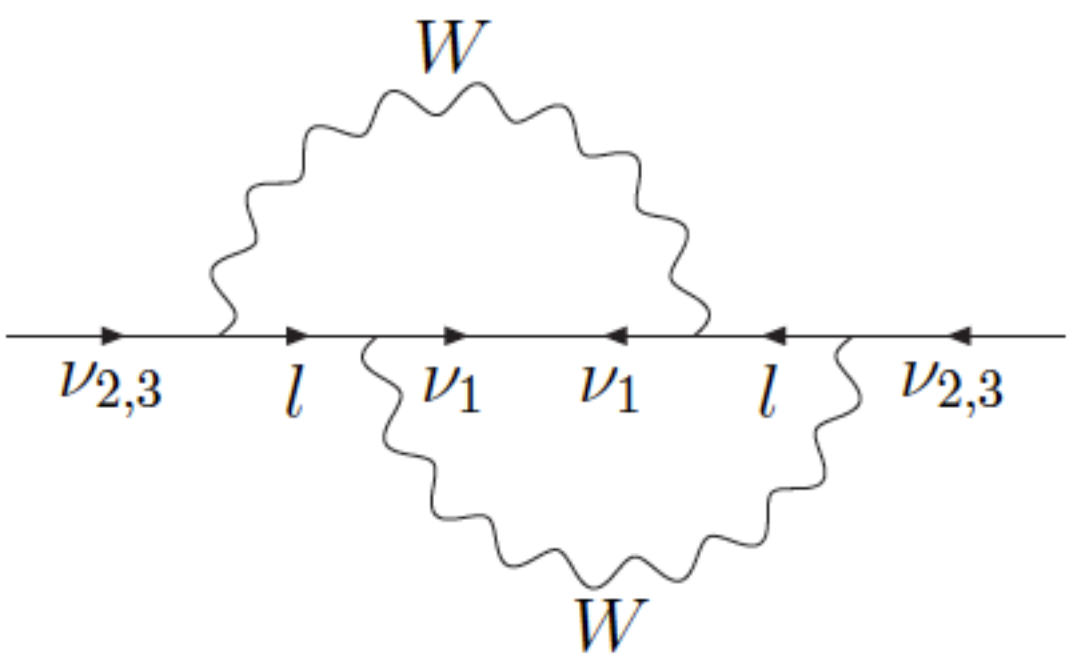
$$\delta m_H^2 = -3\frac{y_\Sigma^\dagger y_\Sigma}{16\pi^2}\left[2\,\Lambda^2 + 2\,M_\Sigma^2 \log\frac{M_\Sigma^2}{\Lambda^2}\right]$$

seesaw type III

RADIATIVE MODELS

[K. S. Babu, E. Ma, Phys. Rev. Lett. 61, 674 (1988)]

add only one N_R so only ν_1 gets mass @ tree-level



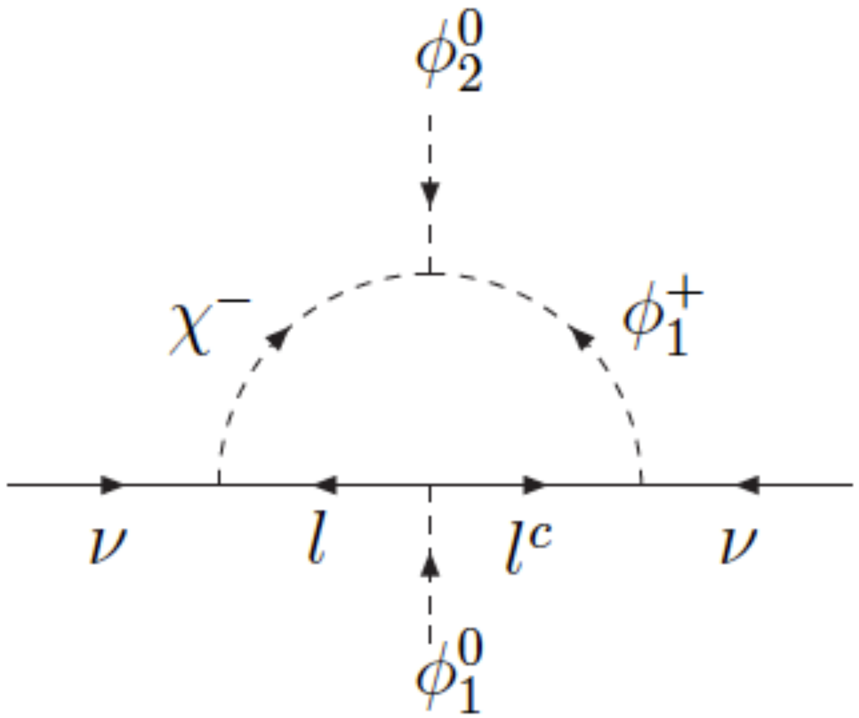
Two-loop origin of neutrino mass

Doubly suppressed by GIM ➔ masses too small!

RADIATIVE MODELS

[A. Zee, Phys. Lett. 93B, 389 (1980)]

Φ_1, Φ_2 = scalar doublets χ = charged scalar singlet



Φ_2 does not couple to leptons

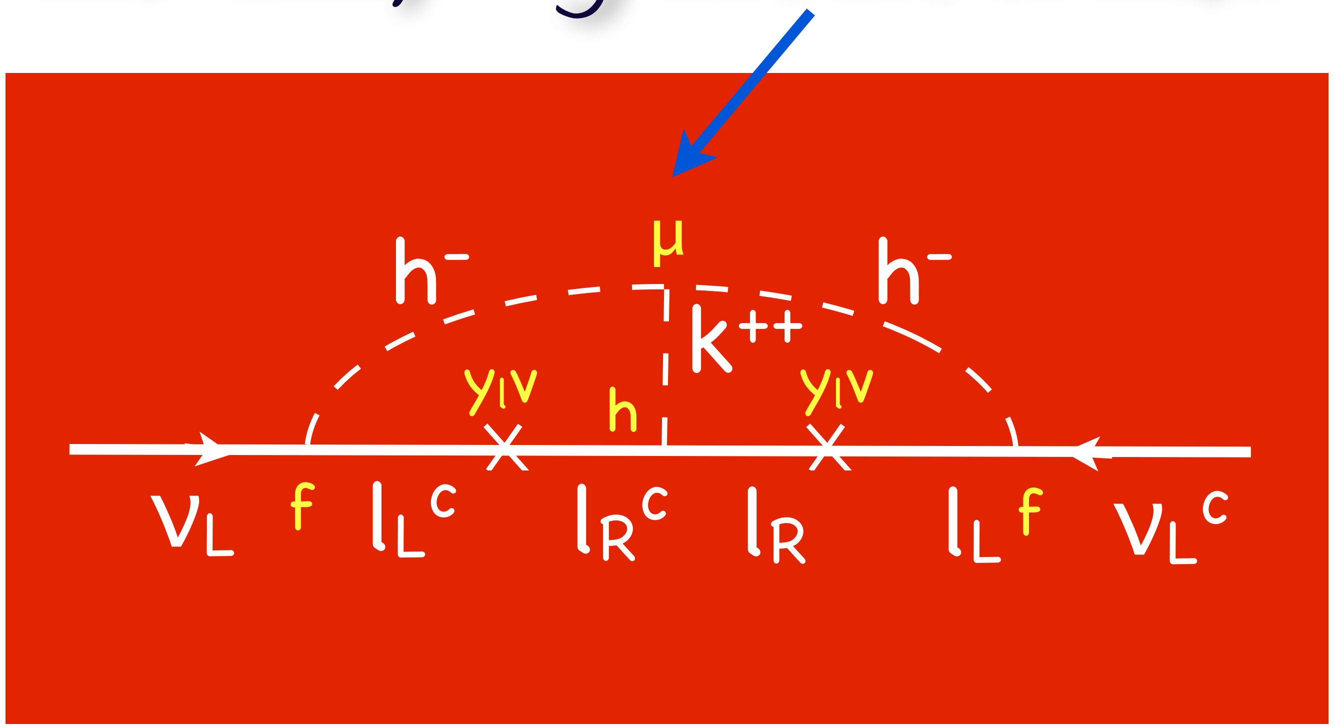
One-loop origin of neutrino mass

Ruled-out by data ...

RADIATIVE MODELS

[A. Zee, Phys. Lett. B93, 389 (1980); K. S. Babu, Phys. Lett. B203, 132 (1988); J. T. Peltoniemi and J. W. F. Valle, Phys. Lett. B304, 147 (1993)]

trilinear coupling which breaks B-L



k^{++} = doubly-charged scalar

h^- = singly-charged scalar

double suppressed by lepton mass

Two-loop origin of neutrino mass

$$\mathbf{M_M \sim \mu\, f\, y_l\, h\, y_l\, f^T\, v^2\, I}$$

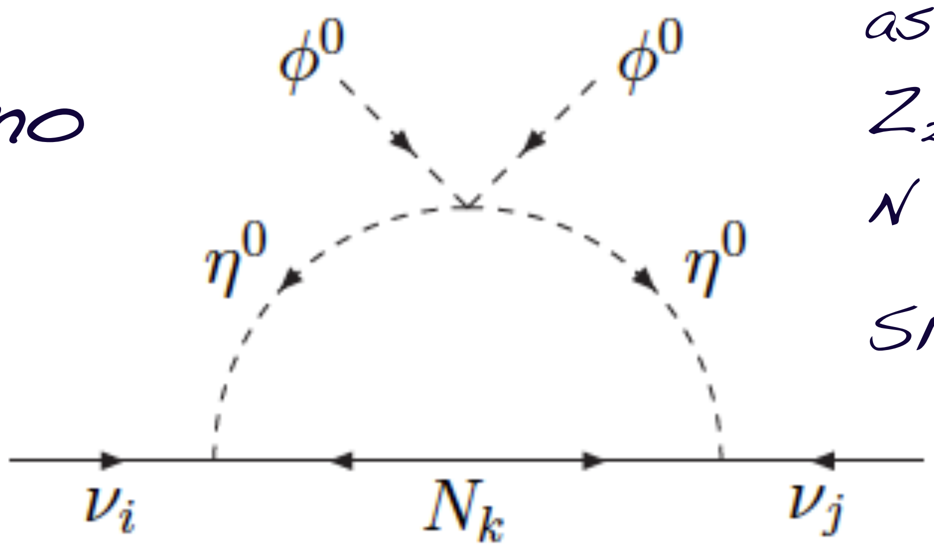
Not yet ruled-out by data ...

RADIATIVE MODELS

[E. Ma Phys. Rev. D73, 077301 (2006)]

add 3 N_R's + a new scalar doublet η

scotogenic neutrino



assume new conserved Z_2 symmetry :

N and η odd under Z_2

SM particles even under Z_2

One-loop origin of neutrino mass

$(\nu\phi^0 - l\phi^+)N$ ✗

$(\nu\eta^0 - l\eta^+)N$ ✔

Not yet ruled-out by data ...

SUSY & R-parity Violation

[see Y. Grossman and S. Rakshit, Phys. Rev. D 69, 093002 (2004) and Ref. therein]

Bilinear R-parity violation

- The general R parity violating superpotential is

$$\frac{1}{2}\lambda_{ijk}\ell_i\ell_j\bar{e}_k + \lambda'_{ijk}\ell_i q_j \bar{d}_k + \frac{1}{2}\lambda''_{ijk}\bar{u}_i\bar{d}_j\bar{d}_k + \epsilon_i H_u L_i$$

⇨ We assume that $\lambda = \lambda' = \lambda'' = 0$. This requires either an additional symmetry or spontaneous R parity breaking.

- The superpotential in bilinear R parity violating models is Nowakowski, Pilaftis, Joshipura, Valle, Romão, Grossman, Nir, Nardi, Banks, Nilles, Polonsky, ...

$$W = \varepsilon_{ab}\left[h_U^{ij}\widehat{Q}_i^a\widehat{U}_j\widehat{H}_u^b + h_D^{ij}\widehat{Q}_i^b\widehat{D}_j\widehat{H}_d^a + h_E^{ij}\widehat{L}_i^b\widehat{R}_j\widehat{H}_d^a - \mu\widehat{H}_d^a\widehat{H}_u^b + \epsilon_i\widehat{L}_i^a\widehat{H}_u^b\right]$$

- The relevant bilinear terms in the soft supersymmetry breaking sector are

$$V_{\text{soft}} = m_{H_u}^2 H_u^{a*}H_u^a + m_{H_d}^2 H_d^{a*}H_d^a + M_{L_i}^2\widetilde{L}_i^{a*}\widetilde{L}_i^a - \varepsilon_{ab}\left(B\mu H_d^a H_u^b + B_i\epsilon_i\widetilde{L}_i^a H_u^b\right)$$

EWSB ⇨ H_d, H_u and sneutrinos acquire vev

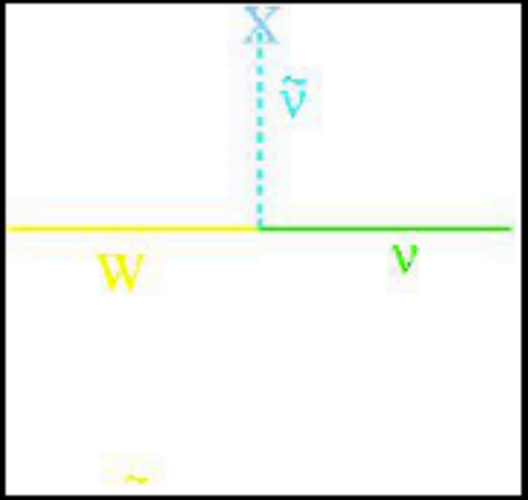
SUSY & R-parity Violation

- sneutrino vev's contributes to the mixing between neutrinos and neutralinos. In the basis $\psi^{0T} = (-i\lambda', -i\lambda^3, \widetilde{H}_d^1, \widetilde{H}_u^2, \nu_e, \nu_\mu, \nu_\tau)$ the neutral fermion mass matrix is

$$M_N = \begin{bmatrix} \mathcal{M}_{\chi^0} & m^T \\ m & 0 \end{bmatrix} \quad \text{where} \quad m = \begin{bmatrix} -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & \epsilon_1 \\ -\frac{1}{2}g'v_2 & \frac{1}{2}gv_2 & 0 & \epsilon_2 \\ -\frac{1}{2}g'v_3 & \frac{1}{2}gv_3 & 0 & \epsilon_3 \end{bmatrix}$$

- For $|\epsilon_i| \ll \mu$ we define $\xi \equiv m \cdot \mathcal{M}_{\chi^0}^{-1}$. M_N is approximately diagonalized by

$$\mathcal{N}^* \simeq \begin{pmatrix} 1 & \xi^\dagger \\ -\xi & 1 \end{pmatrix},$$



$$\mathrm{M}^{\mathrm{eff}} = -m\,\mathcal{M}_{\chi^0}^{-1}\,m^T = \frac{M_1 g^2 + M_2 g'^2}{4\,\det(M_{\chi^0})} \begin{pmatrix} \Lambda_1^2 & \Lambda_1\Lambda_2 & \Lambda_1\Lambda_3 \\ \Lambda_1\Lambda_2 & \Lambda_2^2 & \Lambda_2\Lambda_3 \\ \Lambda_1\Lambda_3 & \Lambda_2\Lambda_3 & \Lambda_3^2 \end{pmatrix}$$

with $\Lambda_i = \mu v_i + v_d \epsilon_i$. This is a low scale see-saw!

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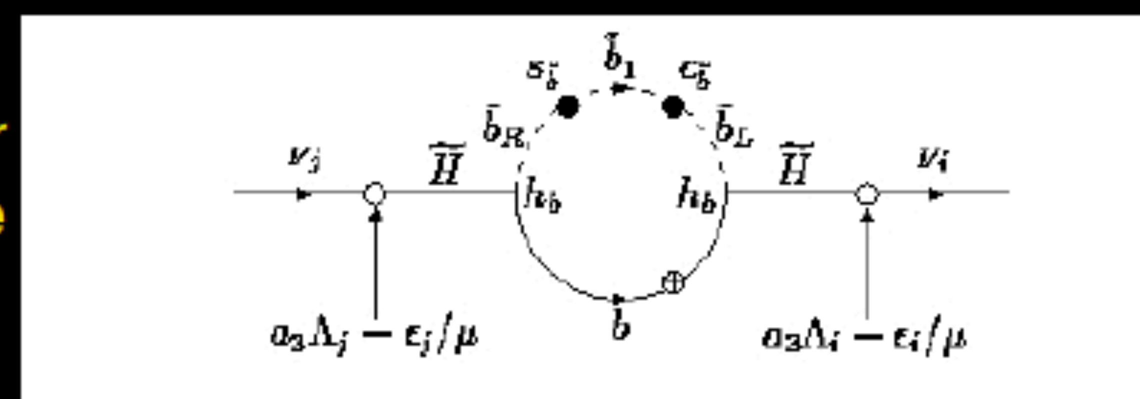
with $\Lambda_i = \mu v_i + v_d \epsilon_i$. This is a low scale see-saw!

- M^{eff} exhibits just one massive neutrino at tree level:

$$m_{\nu_3}^{\mathrm{tree}} = \frac{M_1 g^2 + M_2 g'^2}{4\,\det(M_{\chi^0})}|\vec{\Lambda}|^2 \;, \quad \tan\theta_{13} = -\frac{\Lambda_1}{\sqrt{\Lambda_2^2 + \Lambda_3^2}} \quad \text{and} \quad \tan\theta_{23} = -\frac{\Lambda_2}{\Lambda_3}\,.$$

- The inclusion of radiative correction for the neutral fermion mass matrix gives rise to masses to the three neutrinos.



⇨ only one massive neutrino @ tree-level

⇨ inclusion of radiative corrections give rise to masses to all 3 neutrinos

B-L SPONTANEOUSLY BROKEN

[Y. Chikashige, R.N. Mohapatra, RD> Peccei, Phys. Lett. B98, 265 (1981); PRL 45, 1926 (1980)]

introduce a gauge singlet scalar S

L=2

$$\mathcal{L}_y = -\sum_{l,l'} a_{ll'}\, \overline{N}^c_{lR}\, N_{l'R}\, S + h.c.$$

$\langle S \rangle$ acquires a vev

global $\not{U}_{B-L}$

✔ $M_{ll} = a_{ll} \langle S \rangle$ Majorana Mass

✔ J Majoron (Goldstone Boson)

Extra Flat Dimensions

[K.R. Dienes et al., Nucl. Phys. B557, 25 (1999); N. Arkani-Hamed et al., Phys. Rev D65, 024032 (2002); G.R. Dvali and A.Yu. Smirnov, Nucl. Phys. B563, 63 (1999); R. N. Mohapatra et al., Phys. Lett. B 466, 115 (1999); R.N. Mohapatra and A. Perez-Lorenzana, Nucl. Phys. B576, 466 (2000)]

$\delta = 1$

SM particles propagate in the 3-D brane

3 families of SM singlets propagate in the 4-D bulk

$$S = \int d^4x\, dy\, i\Psi^\alpha \Gamma_J \partial^J \Psi^\alpha + \int d^4x\, (i\bar{\nu}_L^\alpha \gamma_\mu \partial^\mu \nu_L^\alpha + \lambda_{\alpha\beta} H \bar{\nu}_L^\alpha \Psi_R^\beta(x,0) + h.c.)$$

SM flavor neutrinos → ν_L^α

SM singlet bulk fermion fields → Ψ_R^β

Yukawa couplings to SM H

$$\lambda_{\alpha\beta} = h_{\alpha\beta}/\sqrt{M^*}$$

$\Gamma_J\ J = 0, \ldots, 4$: the 5D Dirac γ matrices

$$\mathbf{M_{Pl}^2} = (\mathbf{M}^*)^{\delta+\mathbf{2}}\, \mathbf{V}_\delta$$

One can decompose Ψ^α(x,y) in KK-modes

a = radius of the largest compact extra dimension

$$\Psi^\alpha(x,y) = \frac{1}{\sqrt{2\pi a}} \sum_{N=-\infty}^{\infty} \Psi^{\alpha(N)}(x)\, e^{iNy/a}$$

Extra Flat Dimensions

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$$\nu^{(0)}_{\alpha R} \equiv \Psi^{(0)}_{\alpha R} \qquad \nu^{(0)}_{\alpha L} \equiv \nu_{\alpha L}$$

$$\nu^{(N)}_{\alpha R,L} \equiv \frac{1}{\sqrt{2}}\left(\Psi^{(N)}_{\alpha R,L} \pm \Psi^{(-N)}_{\alpha R,L}\right) \qquad N = 1, ..., \infty$$

$$m^D_{\alpha\beta} = h_{\alpha\beta}\, v\, M^* / M_{\rm Pl}$$

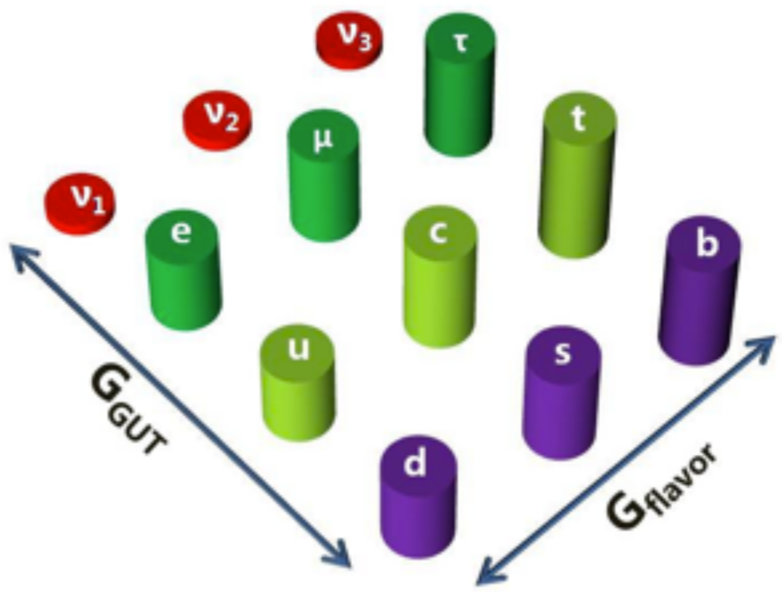
Naturally Small Dirac Mass Term

$$\sum_{\alpha,\beta} m^D_{\alpha\beta}\left[\overline{\nu}^{(0)}_{\alpha L}\,\nu^{(0)}_{\beta R} + \sqrt{2}\,\sum_{N=1}^{\infty}\overline{\nu}^{(0)}_{\alpha L}\,\nu^{(N)}_{\beta R}\right] + \sum_{\alpha}\sum_{N=1}^{\infty}\frac{N}{a}\,\overline{\nu}^{(N)}_{\alpha L}\,\nu^{(N)}_{\alpha R}$$

$$+\frac{g}{\sqrt{2}}\sum_{\alpha}\overline{l_\alpha}\gamma^{\mu}\left(1-\gamma_5\right)\nu^{(0)}_{\alpha}\,W_{\mu} + \text{h.c.}$$

Flavor Models

try to describe the structure of masses



Enforce a G_F symmetry

- non-Abelian
- discrete
- global
- commute w/ gauge group
- spontaneously broken @ high energies

need to extend the scalar sector (Flavon Fields)

G_F = A4, S4, Dn etc..

many models ...

☛ constrains Yukawas : the pattern of mass and mixing

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- ☛ Effective Theory arguments indicate that ν mass is due to Physics Beyond the SM

- ☛ The basic seesaw mechanisms are elegant ways to implement small ν masses in models beyond SM

But there are other ways ...

Consequences of ν masses & mixings

- Decay: $\nu_2 \to \nu_1 \gamma$ happens in all models

  (harmless is m_ν < few eV)

- $\mu \to e \gamma$ happens in all models

  (constrain models where m_ν is of rad. origin)

- Some models have new particles ~ few x 100 GeV / LNV interactions that can be observed @ LHC

- Leptogenesis is possible (next week)