The Physics of Neutrinos

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Lectures :

1. Panorama of Experiments
2. Neutrino Oscillations
3. Models for Neutrino Masses
4. Neutrinos in Cosmology

Lecture IV

Neutrinos in Cosmology

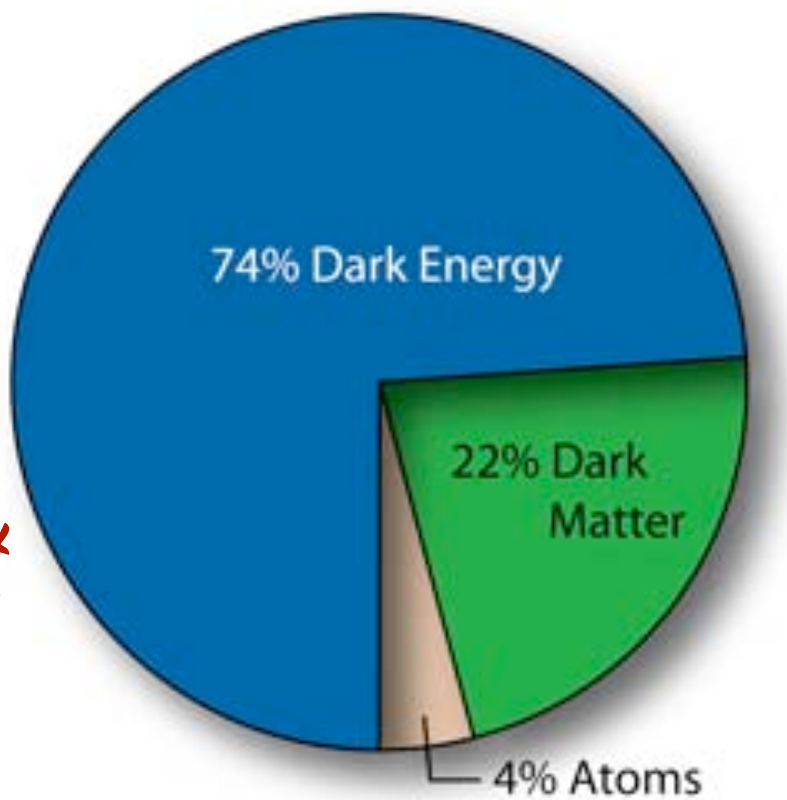
Our Oldest Relics

A Taste of Cosmology

Minimal Cosmological Model: Λ-Cold Dark Matter

Energy-matter content of the Universe (today):

- non-relativistic matter component (26.4%)
- radiation component (0.1 %)
- vacuum energy density and/or cosmological constant (73.5%)



Energy Budget

A Taste of Cosmology

Fundamental Ingredient: Inflation

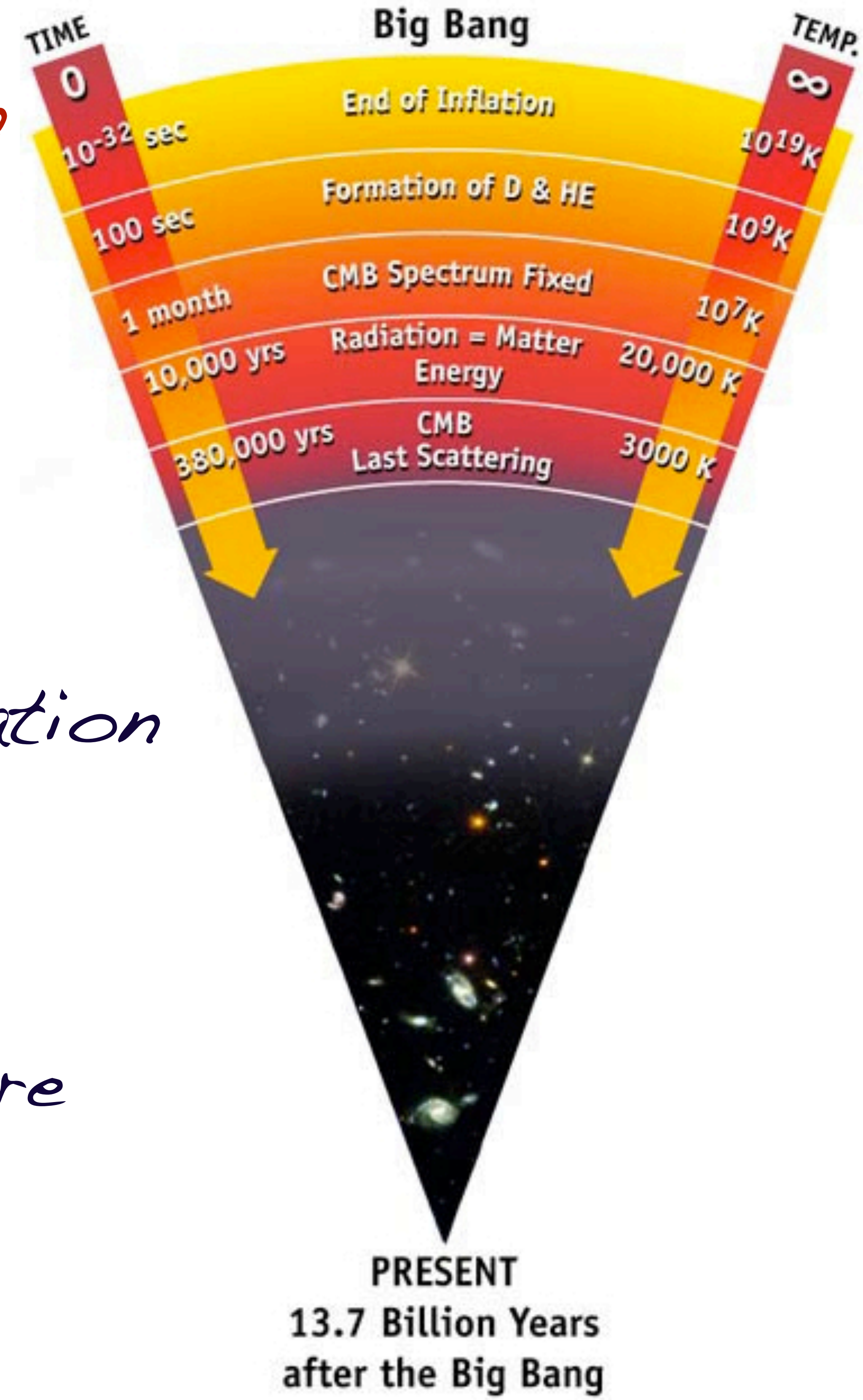
rapid exponential growth ➙

homogeneity, isotropy and

flatness observed in the Universe

perturbations @ the end of Inflation

- act as seeds for formation of Large Scale Structures
- responsible for CMB temperature anisotropies



Relic Neutrinos

ν's : most abundant (known)

particles in the Universe after

CMB γ's

CMB ν : oldest relic

decouple when T~ 1 MeV

$$\mathbf{T}_{\nu \mathbf{0}} = \left(\frac{\mathbf{4}}{\mathbf{11}}\right)^{\mathbf{1/3}} \mathbf{T}_{\mathbf{0}}^{\mathrm{CMB}} = \mathbf{1.945}\,\mathrm{K}\;(1.7\times 10^{-4}\,\mathrm{eV})$$

$$\nu_{\mathbf{e}}, \bar{\nu}_{\mathbf{e}}, \nu_{\mu}, \bar{\nu}_{\mu}, \nu_{\tau}, \bar{\nu}_{\tau}$$

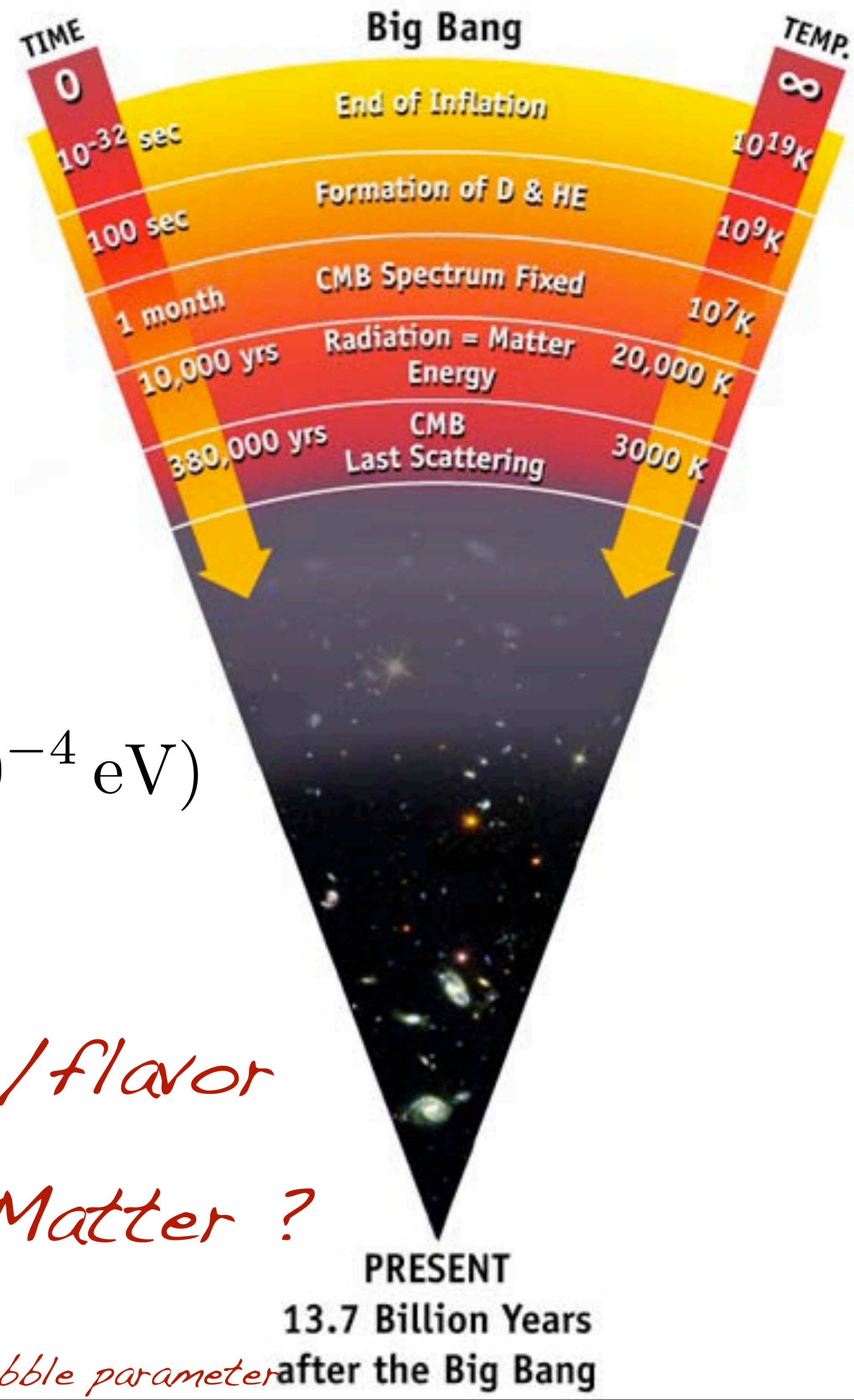
$$\mathbf{n}_{\nu_{\mathbf{i}}\mathbf{0}} \equiv \mathbf{n}_{\nu_{\mathbf{i}}}(\mathbf{T}_{\nu \mathbf{0}}) \approx \mathbf{56}\ \mathrm{cm}^{-3}$$

112 cm^{-3}/flavor

$$\mathbf{\Omega}_{\nu \mathbf{0}} \equiv \frac{\sum_{\mathbf{i}} \mathbf{n}_{\nu_{\mathbf{i}}\mathbf{0}} \mathbf{m}_{\mathbf{i}}}{\rho_{\mathbf{c},\mathbf{0}}\ \mathbf{h}^{\mathbf{2}}} \approx \frac{\sum_{\mathbf{i}} \mathbf{m}_{\mathbf{i}}}{(\mathbf{93}\,\mathbf{eV}\ \mathbf{h}^{\mathbf{2}})}$$

Dark Matter ?

h = present Hubble parameter

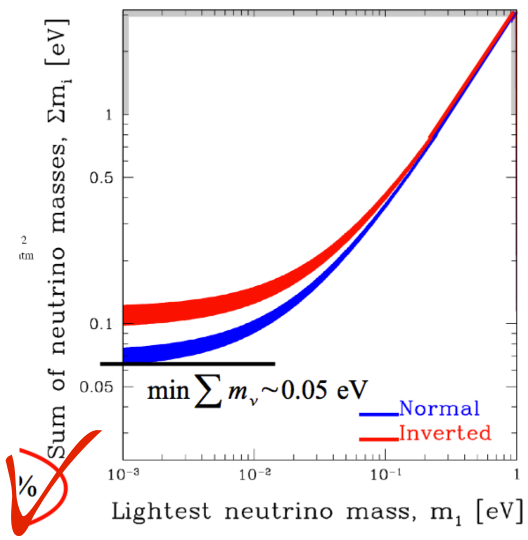


Limits on ν Mass

Neutrino Oscillation Experiments ➡ $\sum m_i \geq 0.05$ eV

Structure Formation ➡ $\Omega_{HDM,0}/\Omega_{DM,0} \leq 0.05$

➡ $\sum m_i \leq 0.6$ eV



So $0.05 \text{ eV} \leq \sum m_i \leq 0.6$ eV

so neutrinos cannot account for all DM but may play another important cosmological role: generate the matter-antimatter asymmetry

Matter-$\overline{\text{Matter}}$ Asymmetry

Baryon Asymmetry of the Universe (BAU)

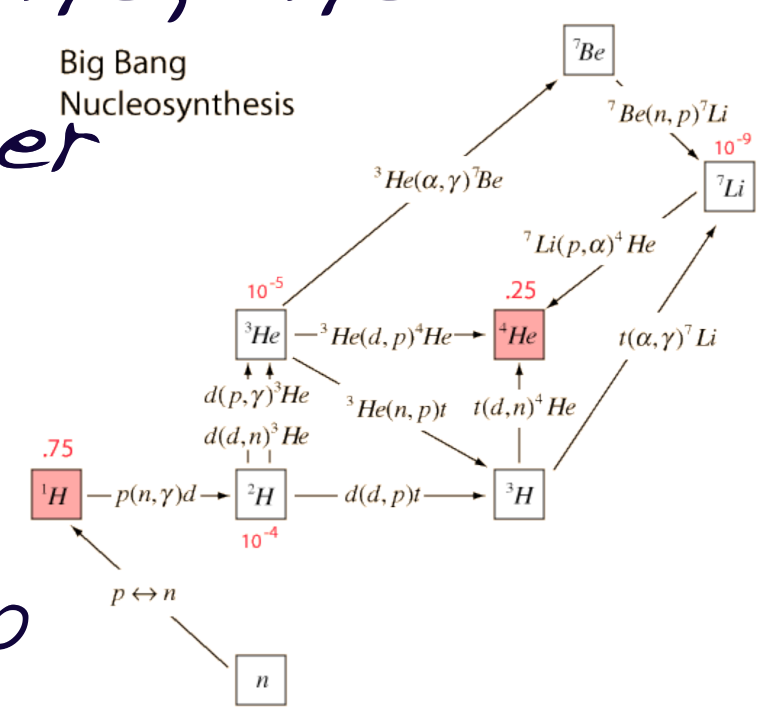
observational evidences establish dominance of matter over antimatter in our Universe

Big Bang Nucleosynthesis (BBN)

in standard BBN predictions on primordial abundances of light elements: D, ^3He, ^4He and ^7Li depend of a single parameter

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma}$$

baryon to photon ratio



Deuterium abundance

$$\eta^D_B = (5.9 \pm 0.5) \times 10^{-10}$$

Baryon Asymmetry of the Universe (BAU)

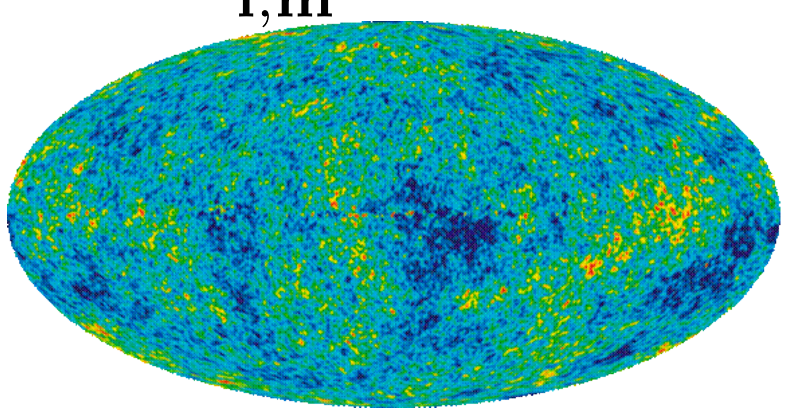
Cosmic Microwave Background (CMB)

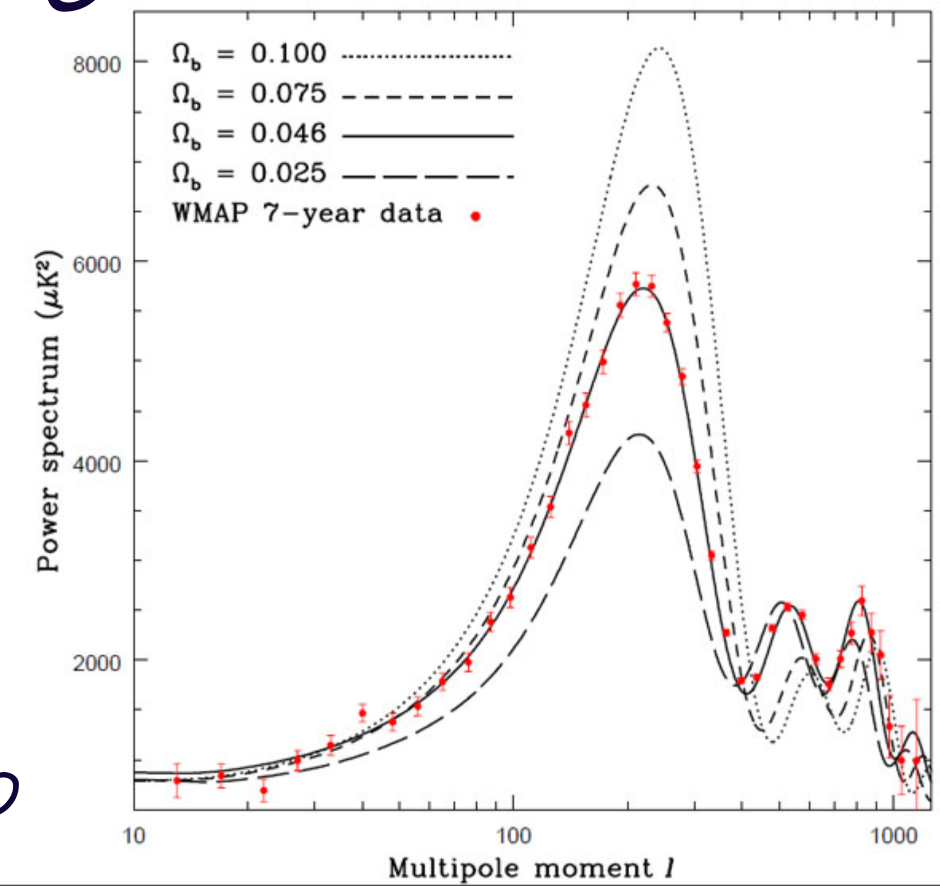
after BBN η_B remains constant so $\eta_B = \eta_{B0}$

this can be related to the baryon energy density $\eta_{B,0} = 274\ \Omega_{B,0} h^2 \times 10^{-10}$

$$\frac{\Delta T}{T} = \sum_{l,m} a_{lm} Y_{ml}(\theta, \phi)$$

$$C_l = \frac{1}{(2l+1)} \sum_m < |a_{lm}|^2 >$$





$$\eta^{CMB}_B = (6.2 \pm 0.15) \times 10^{-10}$$

Baryon Asymmetry of the Universe (BAU)

$\eta^{D}{}_{B} = (5.9 \pm 0.5) \times 10^{-10}$

very good agreement

$\eta^{CMB}{}_{B} = (6.2 \pm 0.15) \times 10^{-10}$

6 orders of magnitude in T !

+ no trace of cosmological antimatter ever observed
+ small amount of e^+, $\bar{p}$ in cosmic rays

& no anti-nuclei

all matter/antimatter that we observe today have to be produced after Inflation → BAU must be dynamically generated after Inflation and before BBN

BARYOGENESIS

Baryogenesis

Sakharov's Conditions

1967 - Sakharov's 3 basic conditions for a successful baryogenesis

- ★ B violation
- ★ C & CP violation
- ★ deviation from thermal equilibrium

★ B violation

~~B+L~~ in the SM

B and L are anomalous in the SM

$$\mathbf{J}^{\mathbf{B}}_{\mu} = \frac{\mathbf{1}}{\mathbf{3}} \left(\overline{\mathbf{Q}}_{\alpha}\gamma_{\mu}\mathbf{Q}_{\alpha} + \overline{\mathbf{U}}_{\alpha}\gamma_{\mu}\mathbf{U}_{\alpha} + \overline{\mathbf{D}}_{\alpha}\gamma_{\mu}\mathbf{D}_{\alpha}\right)$$

$$\mathbf{J}^{\mathbf{L}}_{\mu} = \left(\overline{\mathbf{L}}_{\alpha}\gamma_{\mu}\mathbf{L}_{\alpha} + \overline{\mathbf{E}}_{\alpha}\gamma_{\mu}\mathbf{E}_{\alpha}\right)$$

B and L currents not conserved @ 1-loop

$$\partial^{\mu}\mathbf{J}^{\mathbf{L}}_{\mu} = \partial^{\mu}\mathbf{J}^{\mathbf{B}}_{\mu}$$

$$= \frac{\mathbf{N_f}}{\mathbf{32}\pi^{\mathbf{2}}} \left(-\mathbf{g}^{\mathbf{2}}\,\mathbf{W}^{\mathbf{i}}_{\mu\nu}\widetilde{\mathbf{W}}^{\mathbf{i}\mu\nu} + \mathbf{g}^{'\mathbf{2}}\,\mathbf{B}_{\mu\nu}\widetilde{\mathbf{B}}^{\mu\nu}\right) \quad \text{@ 1-loop}$$

$\mathbf{N_f}$: # generations

$\mathbf{W}$: $SU(2)_L$ field strength

$\mathbf{B}$: $U(1)_Y$ field strength

$\mathbf{g}$: $SU(2)_L$ coupling

$\mathbf{g}'$: $U(1)_Y$ coupling

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$$\mathbf{J}^{\mathbf{L}}_{\mu} = \left(\overline{\mathbf{L}}_{\alpha}\gamma_{\mu}\mathbf{L}_{\alpha} + \overline{\mathbf{E}}_{\alpha}\gamma_{\mu}\mathbf{E}_{\alpha}\right)$$

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generations ($\mathbf{N_f}$)

$SU(2)_L$ field strength ($\mathbf{W}$); $U(1)_Y$ field strength ($\mathbf{B}$)

$SU(2)_L$ coupling ($\mathbf{g}$); $U(1)_Y$ coupling ($\mathbf{g}'$)

B-L conserved

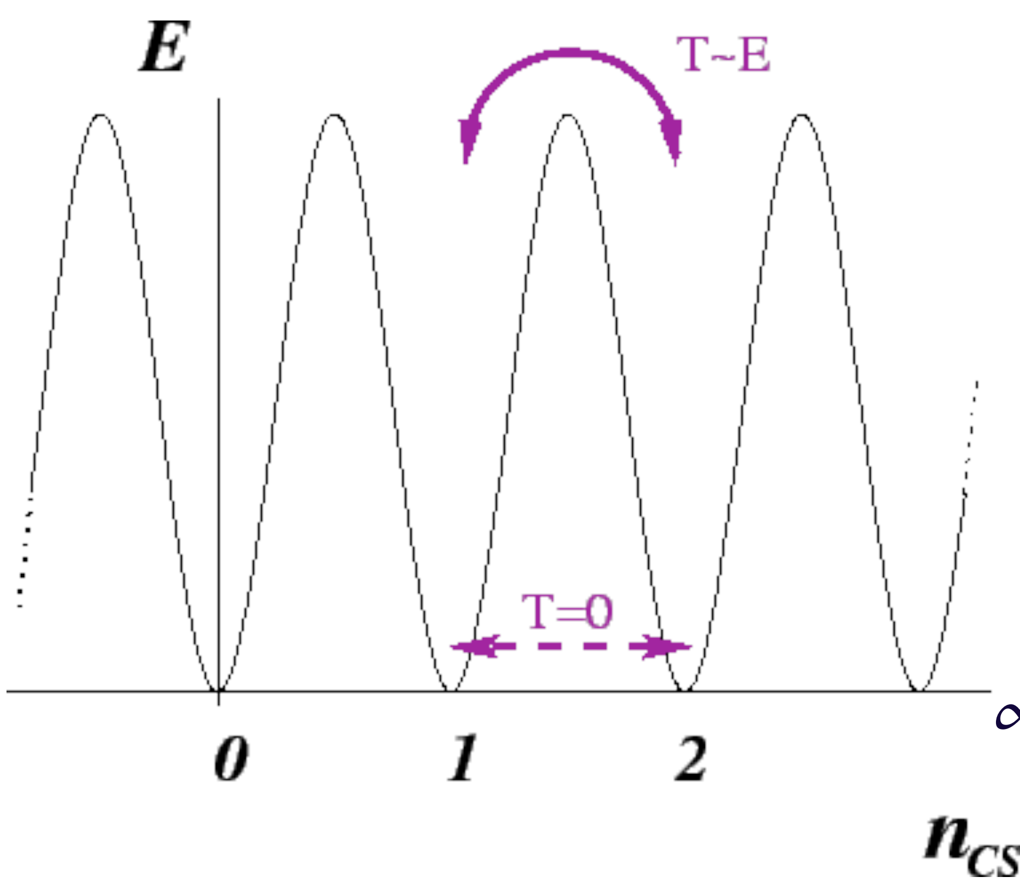
$$\partial^{\mu}(\mathbf{J}^{\mathbf{B}}_{\mu} - \mathbf{J}^{\mathbf{L}}_{\mu}) = \mathbf{0}$$

B+L violated

$$\partial^{\mu}(\mathbf{J}^{\mathbf{B}}_{\mu} + \mathbf{J}^{\mathbf{L}}_{\mu}) \neq \mathbf{0}$$

~~B+L~~ in the SM

Due to the vacuum structure of non-abelian theories B and L are related to change in topological charges of the gauge field (Chern-Simons #)



∞ degenerate ground states

$SU(2)_L$

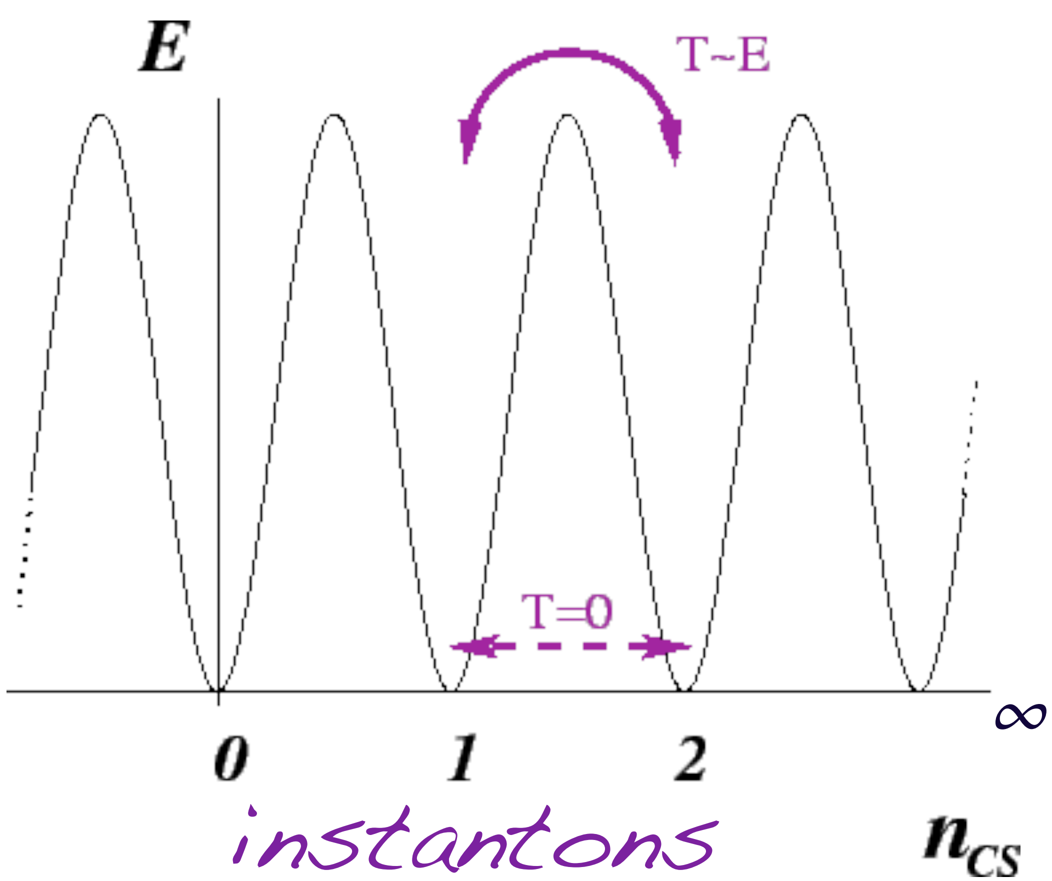
integer

$$\mathbf{B(t_f) - B(t_i) = \int_{t_i}^{t_f} dt \int \partial^\mu J_\mu^B = N_f[n_{CS}(t_f) - n_{CS}(t_i)]}$$

~~B+L~~ in the SM

$$\Delta L = \Delta B = N_f \, \Delta n_{cs} = 3 \, \Delta n_{cs}$$

$$\Delta L = \Delta B = \pm 3$$ minimum jump in the SM



instantons

vacuum ⇒ vacuum transitions

by tunneling the potential barrier

∞ degenerate ground states

$SU(2)_L$ instantons lead to effective operator

$$\mathcal{O}_{B+L} = \Pi_i \, Q_i Q_i Q_i L_i$$

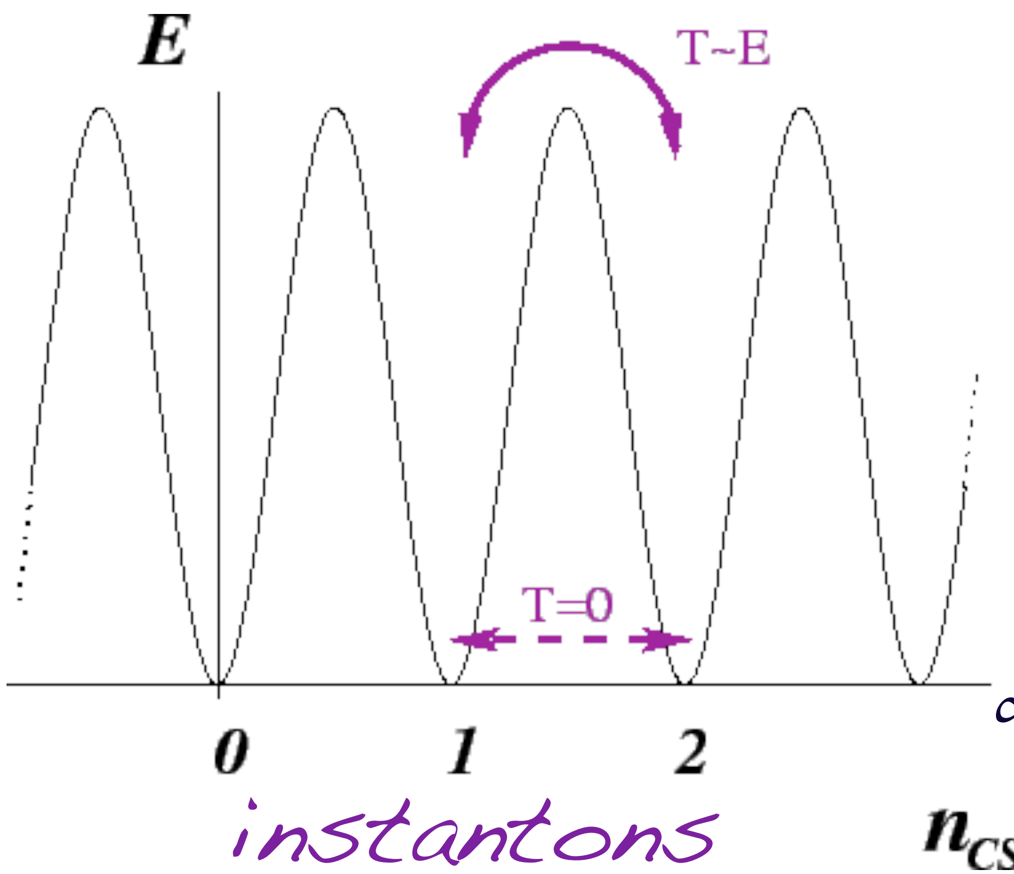
12-fermion interaction

~~B+L~~ in the SM

$\mathcal{O}_{B+L} = \Pi_i \, Q_i Q_i Q_i L_i$ describe processes w/ $\Delta L = \Delta B = \pm 3$

transition rate $\Gamma \sim e^{-S_{inst}} = e^{-4\pi/\alpha} = \mathcal{O}(10^{-165})$

[t'Hooft (76)]



vacuum ⇒ vacuum transitions

by tunneling the potential barrier

∞ degenerate ground states

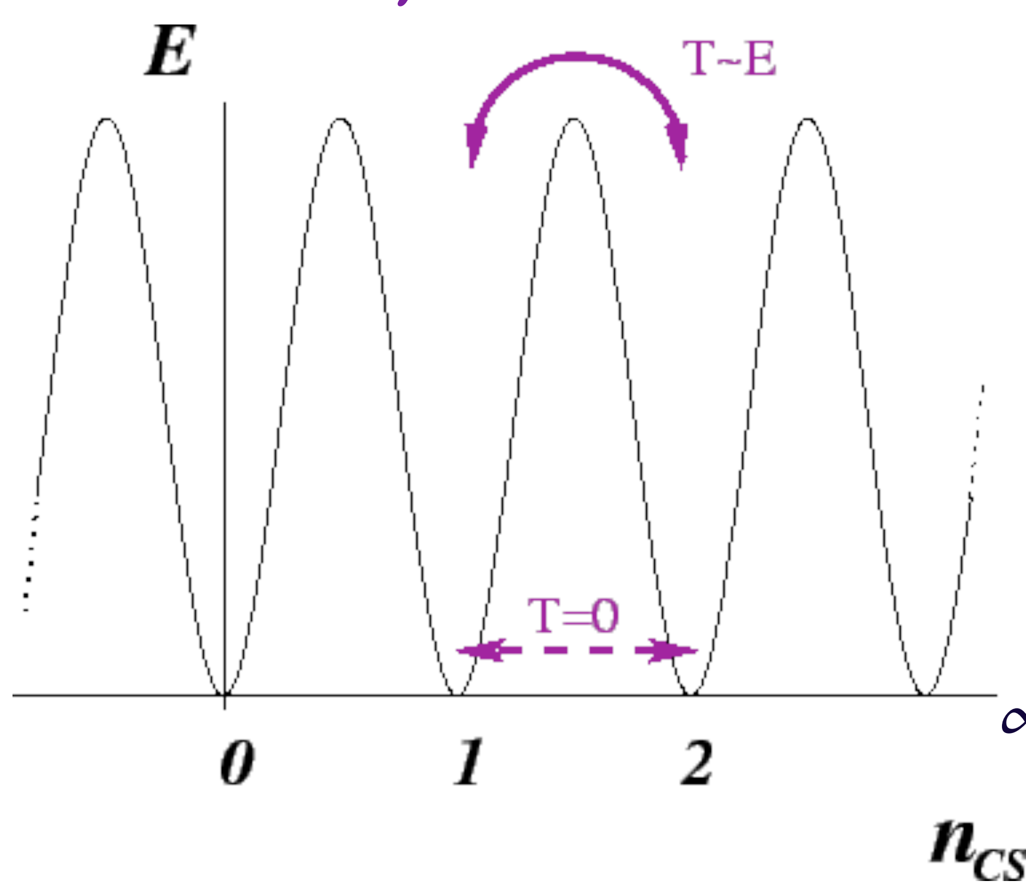
extremely suppressed - negligible in the SM

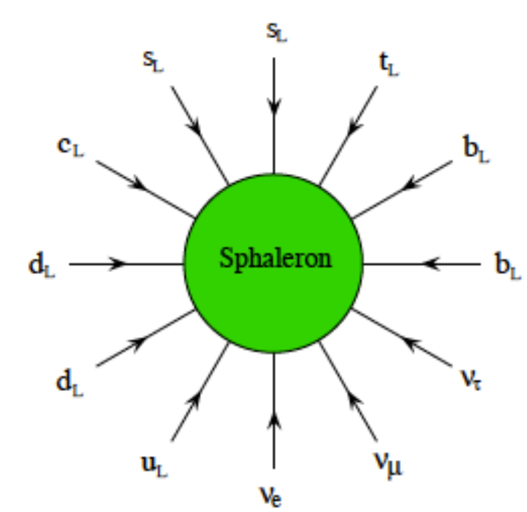
~~B+L~~ in the SM

But in a thermal bath ($T \neq 0$) there can be transitions due to thermal fluctuations (over the barrier)

[Kuzmin, Rubakov, Shaposhnikov (85)]

sphalerons





∞ degenerate ground states

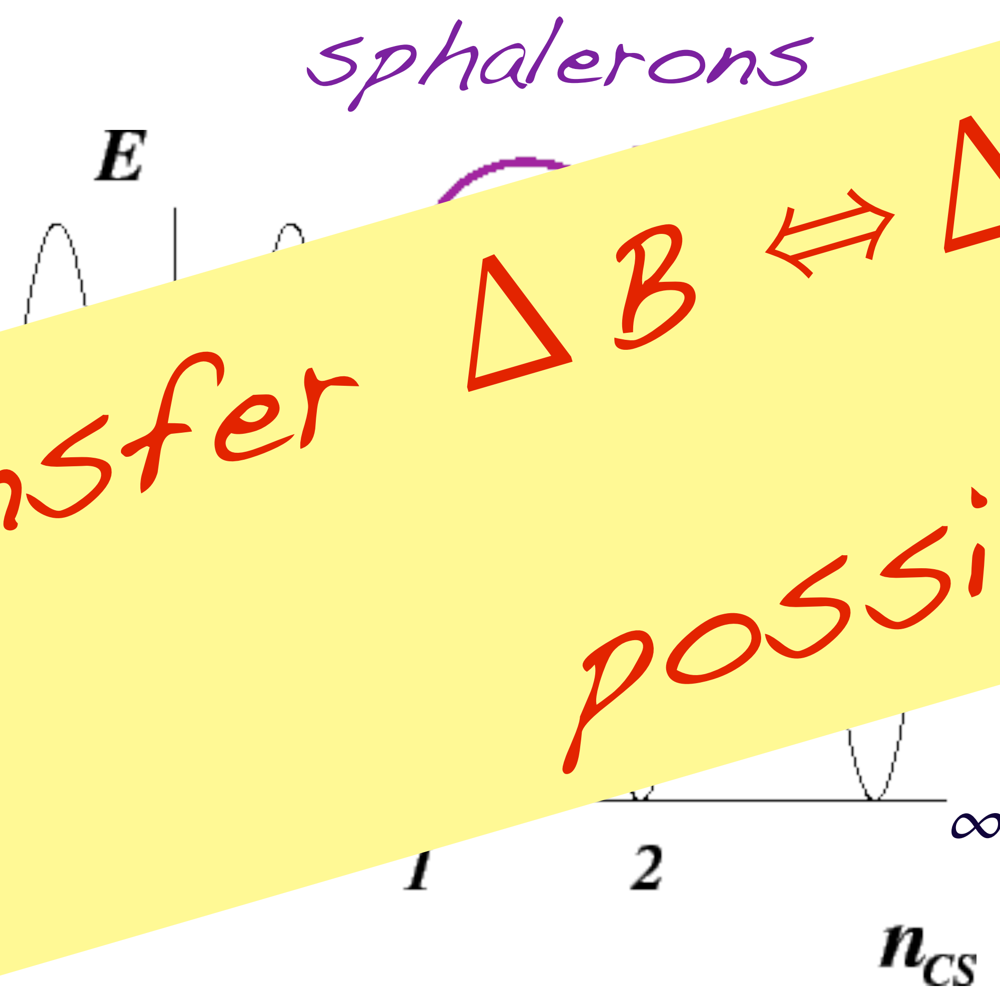
$T > E_{sph}(T)$ no Boltzmann suppression $\quad \Gamma_{B+L} \sim 25\,\alpha^5\,T^4$

100 GeV $< T <$ 10^{12} GeV : ~~B+L~~ rates are in equilibrium

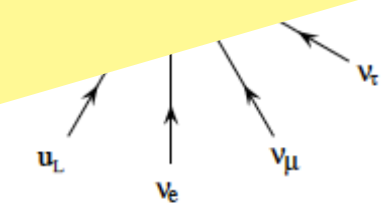
~~B+L~~ in the SM

But in a thermal bath ($T \neq 0$) there can be transitions due to thermal fluctuations (over the barrier)

[Kuzmi



Transfer $\Delta B \Leftrightarrow \Delta L$ becomes possible



∞ degenerate ground states

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Relation between B and L Asymmetries

in a weakly coupled plasma one can assign a chemical potential (μ) to quarks, leptons, Higgs

SM : $5N_f + 1 = 16$ μ_i's

corresponding to the partition function

$$\mathbf{Z}(\mu, \mathbf{T}, \mathbf{V}) = \mathbf{Tr}[\mathbf{e}^{-\beta(\mathbf{H} - \sum_i \mu_i \mathbf{Q}_i)}] \qquad \beta = \mathbf{1}/\mathbf{T}$$

$$\mathbf{\Omega}(\mu, \mathbf{T}) = -\frac{\mathbf{T}}{\mathbf{V}} \ln \mathbf{Z}(\mu, \mathbf{T}, \mathbf{V})$$ thermodynamical potential

$\beta\mu_i << 1$ particle-antiparticle number density asymmetry

$$\mathbf{n_i} - \bar{\mathbf{n}}_i = -\frac{\partial \mathbf{\Omega}(\mu, \mathbf{T})}{\partial \mu_i} = \frac{1}{6} \mathbf{g_i} \mathbf{T}^3 \begin{cases} \beta\mu_i + \mathcal{O}((\beta\mu_i)^3) & \text{fermions} \\ 2\beta\mu_i + \mathcal{O}((\beta\mu_i)^3) & \text{bosons} \end{cases}$$

Distribution of Particles

particle species i in equilibrium

$$\mathbf{f^{eq}_{i,\pm}(p) = \frac{1}{e^{(E_i-\mu_i)/T} \pm 1}}$$

+ fermions

- bosons

equilibrium number density

$$\mathbf{n^{eq}_{i,\pm} = \frac{g_i}{(2\pi)^3}\int d^3p\, f^{eq}_{i,\pm}(p)} \longrightarrow \begin{cases} \dfrac{g_i T^3}{\pi^2}\begin{cases} \zeta(3) + \frac{\mu_i}{T}\zeta(2) + \ldots & \text{bosons} \\ \frac{3}{4}\zeta(3) + \frac{\mu_i}{2T}\zeta(2) + \ldots & \text{fermions} \end{cases} & \mathbf{m_i << T} \\ n^{eq}_{i,MB} & \mathbf{m_i >> T} \end{cases}$$

$$\zeta(\mathbf{2}) = \frac{\pi^{\mathbf{2}}}{\mathbf{6}}, \quad \zeta(\mathbf{3}) = \mathbf{1.202}$$

internal degrees of freedom $\quad \mathbf{g_i = g_{\bar{i}}}$

$$\mathbf{g_U = g_D = g_E = 1} \qquad \mathbf{g_\Phi = g_L = g_Q = 2}$$

Relation between B and L Asymmetries

quarks, leptons and Higgs interact via Yukawa + gauge couplings + non-perturbative sphaleron processes

$$\mathbf{n_B - n_{\overline{B}} = \frac{g_{eff}\,\Delta B T^2}{6}}$$ baryon # density

$$\mathbf{n_{L_i} - n_{\overline{L}_i} = \frac{g_{eff}\,\Delta L_i T^2}{6}}$$ lepton # density

$$\mathbf{\Delta B = \sum_i (2\mu_{q_i} + \mu_{u_i} + \mu_{d_i})} \quad \mathbf{\Delta L = \sum \Delta L_i = \sum_i (2\mu_{l_i} + \mu_{e_i})}$$

thermal equilibrium → relations among μ_i's

$$\mathbf{\Delta B = c_s\,\Delta(B-L)} \quad \mathbf{\Delta L = (c_s - 1)\,\Delta(B-L)} \quad \mathbf{c_s = \frac{8\,N_f + 4}{22\,N_f + 13}}$$

(B-L) @ end of Leptogenesis will determine B asymmetry today

Relation between B and L Asymmetries

$$\mathbf{\Delta B = c_s\, \Delta(B - L)} \qquad\qquad \mathbf{\Delta L = (c_s - 1)\, \Delta(B - L)}$$

$$\mathbf{c_s = \frac{8\,N_f + 4}{22\,N_f + 13}}$$

$N_f = 3 \rightarrow c_s = 28/79 \approx 1/3$

so $\rightarrow \Delta B \approx 1/3\ \Delta(B-L)$

$\Delta L \approx -2/3\ \Delta(B-L)$

(B-L) @ end of Leptogenesis will determine B asymmetry today

★ C & CP violation

Need for ~~C~~ and ~~CP~~

if C and CP conserved → processes involving baryon would have the same rate of processes involving anti-baryons (not net ΔB)

SM:

Both Provided by Weak Interactions

However CP violation is way too small

(by ~10 orders of magnitude ...)

★ deviation from thermal equilibrium

Departure from Thermal Equilibrium

$\hat{C}$

$B \to -B$

$\widehat{CP}$

$B \to -B$

average B in equilibrium at T

$$\langle \mathbf{B} \rangle_{\mathbf{T}} = \mathbf{Tr}[\mathbf{e}^{-\frac{1}{\mathbf{kT}}\mathbf{H}}\mathbf{B}] = \mathbf{Tr}[(\mathbf{CPT})(\mathbf{CPT})^{-1}\mathbf{e}^{-\frac{1}{\mathbf{kT}}\mathbf{H}}\mathbf{B}]$$

$$= \mathbf{Tr}[\mathbf{e}^{-\frac{1}{\mathbf{kT}}\mathbf{H}}(\mathbf{CPT})^{-1}\mathbf{B}(\mathbf{CPT})] = -\mathbf{Tr}[\mathbf{e}^{-\frac{1}{\mathbf{kT}}\mathbf{H}}\mathbf{B}]$$

Imposing $[\mathcal{H}, CPT] = 0$

in equilibrium $\langle B \rangle_T$ vanishes no generation of net B

Departure from Thermal Equilibrium

SM:

departure from thermal equilibrium can happen if at the Electroweak Symmetry Breaking a strong first order phase transition happens

$m_H \leq 45$ GeV

ruled out by LEP

Leptogenesis

Minimal Seesaw Lagrangian: only add R neutrinos to SM

$$\mathcal{L}_{\rm KE} = \imath\,\overline{\mathbf{L}}\,\not{D}\,\mathbf{L} + \imath\,\overline{\mathbf{R}}\,\not{D}\,\mathbf{R} + \imath\,\overline{\mathbf{N}}_{\mathbf{R}}\not{\partial}\,\mathbf{N}_{\mathbf{R}}$$

SM lepton doublets   SM lepton singlets   R neutrino singlets

New Physics Scale

$$\mathcal{L}_{\rm Y} = -\overline{\mathbf{L}}\,\mathbf{\Phi}\,\mathbf{y}_{\ell}\,\mathbf{R} - \overline{\mathbf{L}}\,\tilde{\mathbf{\Phi}}\,\mathbf{y}_{\mathbf{N}}^{\dagger}\,\mathbf{N}_{\mathbf{R}} - \frac{1}{2}\overline{\mathbf{N}}_{\mathbf{R}}\,\mathbf{M}_{\mathbf{R}}\,\mathbf{N}_{\mathbf{R}}^{\mathbf{c}} + \text{h.c.}$$

EWSB ⇨

$$\mathbf{m}_{\nu} \equiv \frac{\mathbf{g}}{\mathbf{\Lambda}}\,\mathbf{v}^{2} = -\frac{1}{2}\,\mathbf{y}_{\mathbf{N}}^{\mathbf{T}}\,\frac{1}{\mathbf{M}_{\mathbf{R}}}\,\mathbf{y}_{\mathbf{N}}\,\mathbf{v}^{2}$$

Majorana Mass Matrix for light neutrinos

Minimal Seesaw Lagrangian: only add R neutrinos to SM

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SM lepton doublets — SM lepton singlets — R neutrino singlets

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Majorana Mass Matrix for light neutrinos

1. Violates L (B−L)
2. May choose N_R's to make M_R real → y_N will have phases (violates CP)
3. Allows for $N \to L\Phi$ decay out of equilibrium

Fulfill all 3 Sahkarov's Conditions [Fukugita, Yanagita (86)]

Minimal Leptogenesis

1. 3 Heavy RH neutrinos (N_i)

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4. single flavor leptogenesis

5. only N_1 responsible for final asymmetry

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2. $M_1 \ll M_2, M_3$ (hierarchical)
3. N_i produced by Yukawa interactions (thermal leptogenesis) in equilibrium in early Universe
4. single flavor leptogenesis
5. only N_1 responsible for final asymmetry
6. When $T < M_1$, N_1 drops out of equilibrium as it cannot be produced efficiently anymore

Minimal Leptogenesis

7. If $\mathbf{\Gamma(N_1 \to \Phi\, l_\alpha) \neq \Gamma(N_1 \to \Phi^\dagger\, \overline{l_\alpha})}$

$|\Delta L| = 1$

lepton asymmetry will be generated

Minimal Leptogenesis

7. If $\mathbf{\Gamma(N_1 \rightarrow \Phi\, l_\alpha) \neq \Gamma(N_1 \rightarrow \Phi^\dagger\, \overline{l_\alpha})}$

$|\Delta L| = 1$

lepton asymmetry will be generated

8. This asymmetry will be converted to baryon asymmetry by sphaleron processes

Minimal Leptogenesis

7. If $\mathbf{\Gamma(N_1 \to \Phi\, l_\alpha) \neq \Gamma(N_1 \to \Phi^\dagger\, \overline{l_\alpha})}$

$|\Delta L| = 1$

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Baryogenesis through Leptogenesis

Minimal Leptogenesis

To compute baryon asymmetry

1. evaluate CP asymmetry

$$\epsilon_{\mathbf{1}} = \frac{\sum_\alpha \mathbf{\Gamma}(\mathbf{N_1} \to \mathbf{\Phi}\ \mathbf{l}_\alpha) - \mathbf{\Gamma}(\mathbf{N_1} \to \mathbf{\Phi}^\dagger\ \overline{\mathbf{l}_\alpha})}{\sum_\alpha \mathbf{\Gamma}(\mathbf{N_1} \to \mathbf{\Phi}\ \mathbf{l}_\alpha) + \mathbf{\Gamma}(\mathbf{N_1} \to \mathbf{\Phi}^\dagger\ \overline{\mathbf{l}_\alpha})}$$

Minimal Leptogenesis

To compute baryon asymmetry

1. evaluate CP asymmetry

$$\epsilon_1 = \frac{\sum_\alpha \Gamma(N_1 \to \Phi\, l_\alpha) - \Gamma(N_1 \to \Phi^\dagger\, \overline{l_\alpha})}{\sum_\alpha \Gamma(N_1 \to \Phi\, l_\alpha) + \Gamma(N_1 \to \Phi^\dagger\, \overline{l_\alpha})}$$

2. solve Boltzmann Equation to compute washout of the lepton asymmetry

$$s = \frac{g^* 2\pi^2}{45} T^3$$

$$Y_{\Delta L} \approx \frac{n^{eq}_{N_1}}{s} \kappa\, \epsilon_1 = \frac{135\, \zeta(3)}{4\pi^4\, g^*} \kappa\, \epsilon_1$$

$g^* \sim 106$ (SM)

$$n^{eq}_{N_1} = \frac{3}{2} \frac{\zeta(3)}{\pi^2} T^3$$

Minimal Leptogenesis

To compute baryon asymmetry

1. evaluate CP asymmetry

$$\epsilon_1 = \frac{\sum_\alpha \Gamma(N_1 \to \Phi\, l_\alpha) - \Gamma(N_1 \to \Phi^\dagger\, \overline{l_\alpha})}{\sum_\alpha \Gamma(N_1 \to \Phi\, l_\alpha) + \Gamma(N_1 \to \Phi^\dagger\, \overline{l_\alpha})}$$

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$g^* \sim 106$ (SM)

$$n^{eq}_{N_1} = \frac{3}{2}\frac{\zeta(3)}{\pi^2} T^3$$

3. convert lepton asymmetry into Baryon asymmetry

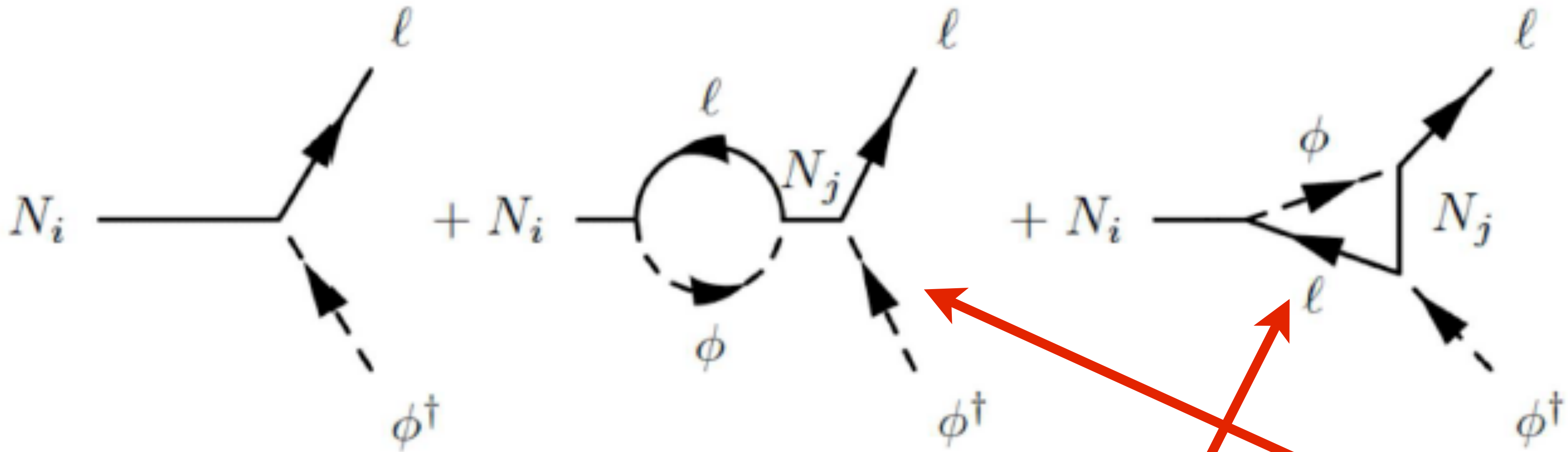
$$Y_{\Delta B} = \frac{n_B - n_{\overline{B}}}{s} = c_s Y_{\Delta(B-L)} = -c_s Y_{\Delta L} \sim (10^{-3} - 10^{-4})\,\epsilon_1$$

CP Asymmetry

At tree-level Γ is real so

$$\Gamma_{\rm tree}(N_1 \to l\,\Phi) = \Gamma_{\rm tree}(N_1 \to \bar{l}\,\Phi^\dagger) = |y|^2\,M_1$$

CP asymmetry comes from interference between tree level and 1-loop contributions



$$\epsilon_1 = \frac{1}{8\pi}\frac{1}{|yy^\dagger|_{11}}\sum_{i=2,3}\mathrm{Im}\left((yy^\dagger)^2_{1i}\right)\left[f\left(\frac{M_i^2}{M_1^2}\right) + g\left(\frac{M_i^2}{M_1^2}\right)\right]$$

$$f(x) = \sqrt{x}\left[1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right] \quad g(x) = \frac{\sqrt{x}}{1-x}$$

CP Asymmetry

For Hierarchical Neutrinos $M_1 << M_2 << M_3$

$$\epsilon_1 = -\frac{3}{8\pi}\frac{1}{|yy^\dagger|_{11}}\sum_{i=2,3}\mathrm{Im}\left((yy^\dagger)^2_{1i}\right)\frac{M_1}{M_i}$$

However if $M_1 \approx M_2$ we can have resonant enhancement of the lepton asymmetry i.e resonant leptogenesis [Pilaftsis, Underwood]

$$\epsilon_1 = -\frac{3}{8\pi}\frac{\sum_j \mathrm{Im}\left((yy^\dagger)^2_{ij}\right)}{|yy^\dagger|_{ii}|yy^\dagger|_{jj}}\frac{(M_i^2 - M_j^2)M_i\Gamma_{N_j}}{(M_i^2 - M_j^2)^2 + M_i^2\Gamma_{N_j}^2}$$

RH neutrino mass scale can be of TeV order

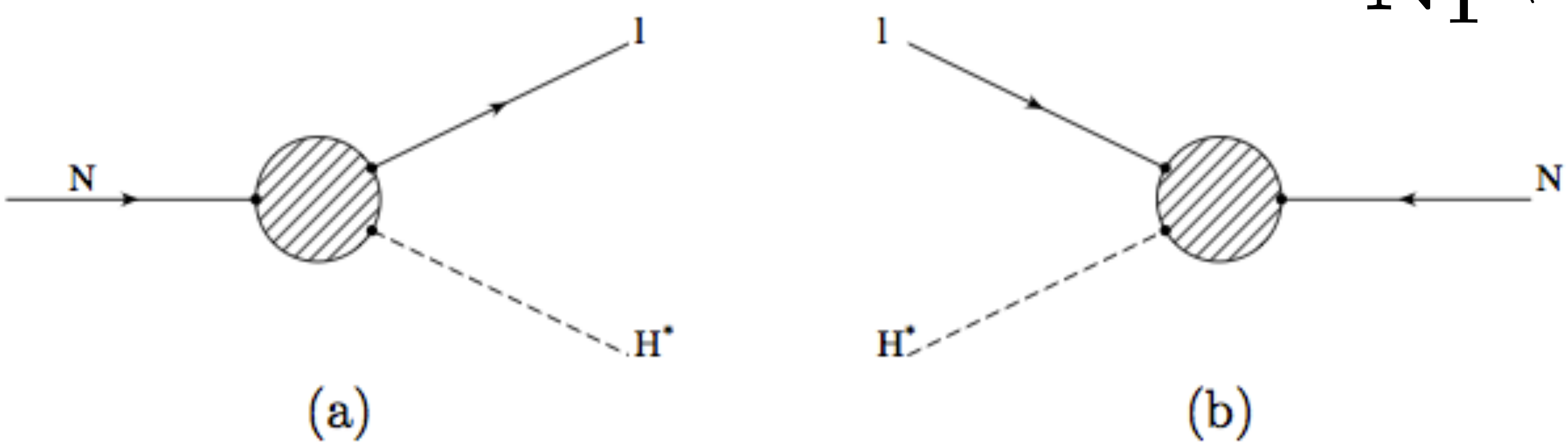
Equilibrium

In equilibrium no net asymmetry

Equilibrium is maintained by

$$\mathbf{N_1} \leftrightarrow \ell + \mathbf{\Phi}$$

$$\mathbf{N_1} \leftrightarrow \bar{\ell} + \mathbf{\Phi}^\dagger$$



Decay

Inverse Decay

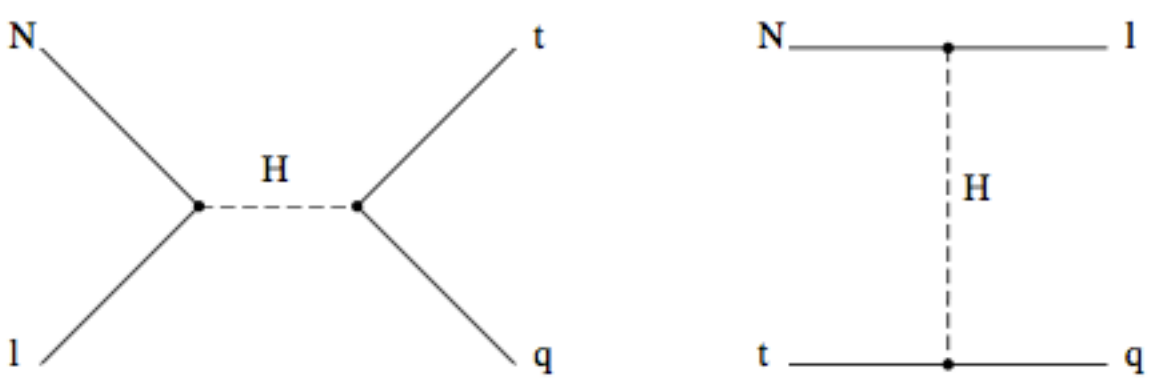
$|\Delta L| = 1$

modify N1 abundance

Equilibrium

In equilibrium no net asymmetry

Equilibrium is maintained by



(a) (b)

2-2 scattering

$|\Delta L| = 1$

$$\mathbf{N_1}\ell \leftrightarrow \mathbf{t\bar{q}} \qquad \mathbf{N_1 t} \leftrightarrow \mathbf{\bar{l}q}$$

$$\mathbf{N_1}\bar{\ell} \leftrightarrow \mathbf{t\bar{q}} \qquad \mathbf{N_1\bar{t}} \leftrightarrow \mathbf{\bar{l}\bar{q}}$$

s-channel t-channel

modify N1 abundance

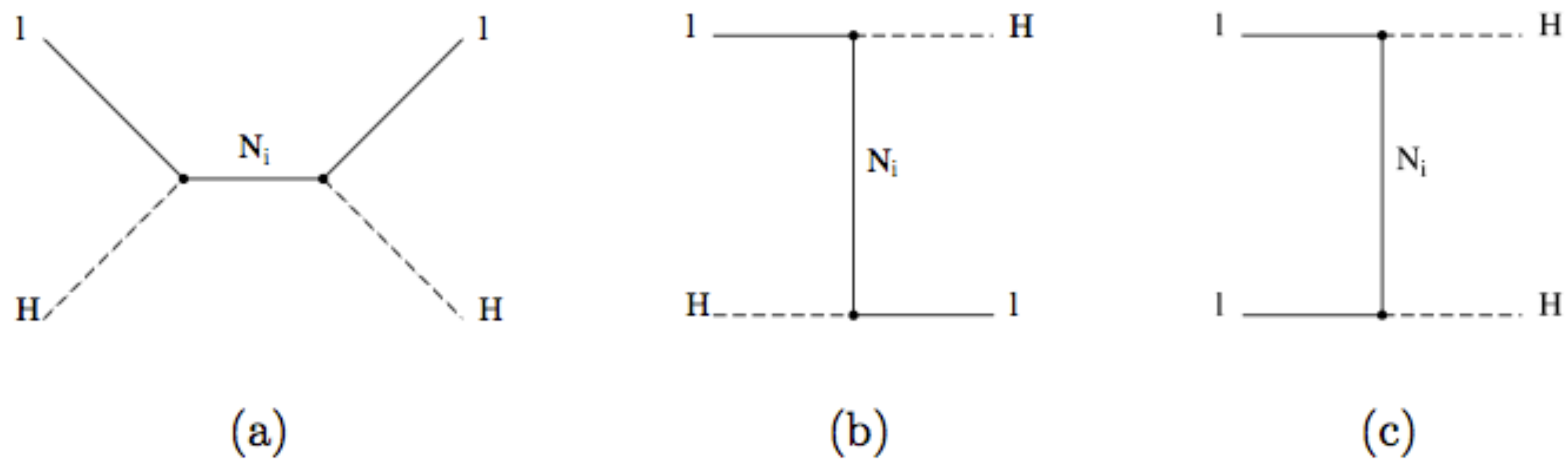
Equilibrium

In equilibrium no net asymmetry

do not modify N1 abundance

Equilibrium is maintained by

2-2 scattering $|\Delta L| = 2$



$$\ell\Phi \to \bar{\ell}\Phi^\dagger$$

$$\ell\ell \to \Phi^\dagger\Phi^\dagger$$

$$\overline{\ell\ell} \to \Phi\Phi$$

Out of Equilibrium

Non-equilibrium is provided by the expansion of the Universe

expansion rate

For $\Gamma(T) < \mathcal{H}(T) \approx 1.7 \sqrt{g^*}\, T^2 / M_{PL}$

Borderline occurs when $\Gamma = \mathcal{H}\,\big|_{T=M_1}$

Production rate

$$\mathbf{\Gamma_{prod}} \sim \sum_\alpha \frac{|\mathbf{y}_{\alpha\mathbf{1}}|^{\mathbf{2}}}{\mathbf{4}\pi}\mathbf{T}$$

Total decay rate

$$\mathbf{\Gamma_D} = \sum_\alpha \mathbf{\Gamma}_{\alpha\alpha} = \sum_\alpha \mathbf{\Gamma}(\mathbf{N_1} \rightarrow \mathbf{\Phi}\ell_\alpha, \mathbf{\Phi}^\dagger\bar{\ell}_\alpha) = \sum \frac{\mathbf{y}^*_{\alpha\mathbf{1}}\mathbf{y}_{\alpha\mathbf{1}}}{\mathbf{8}\pi}\mathbf{M_1}$$

Out of Equilibrium

Define 2 parameters of the order of m_0^ν

$$\widetilde{\mathbf{m}} \equiv \sum_\alpha \widetilde{\mathbf{m}_{\alpha\alpha}} \equiv \sum_\alpha \frac{\mathbf{y}^*_{\alpha 1}\mathbf{y}_{\alpha 1}\mathbf{v}^2}{2\,\mathbf{M}_1} = \frac{8\pi\mathbf{v}^2}{2\,\mathbf{M}_1^2}\Gamma_\mathbf{D}$$

$$\mathbf{m}_* \equiv \frac{8\pi\mathbf{v}^2}{2\,\mathbf{M}_1^2}\mathbf{H}(\mathbf{T}=\mathbf{M}_1) \sim \mathbf{1}\times\mathbf{10}^{-3}\,\mathbf{eV}$$

it can be shown that $\widetilde{\mathbf{m}} > \mathbf{m}_0^\nu$

Washout Scenarios

Strong Washout Scenario $\Gamma_D > H \quad (\tilde{m} > m_*)$

@ $T \approx M_1$ thermal density is obtained $n_{N_1} \sim n_\gamma$

any asymmetry N1 makes is washed out

@ $T < T_\alpha$ N1's decay out of equilibrium

$(\tilde{m} > m_*, \quad \tilde{m}_{\alpha\alpha} > m_*)$

$$\kappa_\alpha \sim \frac{n_{N_1}(T_\alpha)}{n_{N_1}(T >> M_1)} \sim e^{-M_1/T_\alpha} \sim \frac{m_*}{\tilde{m}_{\alpha\alpha}}$$

Washout Scenarios

Strong Washout Scenario $\Gamma_D > H \quad (\tilde{m} > m_*)$

@ $T \approx M_1$ thermal density is obtained $n_{N_1} \sim n_\gamma$

any asymmetry N1 makes is washed out

@ $T < M_1$ N1's start to decay

@ $T < T_\alpha$ N1's decay out of equilibrium

$(\tilde{m} > m_*, \quad \tilde{m}_{\alpha\alpha} > m_*)$

$$\kappa_\alpha \sim \frac{n_{N_1}(T_\alpha)}{n_{N_1}(T >> M_1)} \sim e^{-M_1/T_\alpha} \sim \frac{m_*}{\tilde{m}_{\alpha\alpha}}$$

Washout Scenarios

Strong Washout Scenario $\quad \mathbf{\Gamma_D > H} \quad (\widetilde{\mathbf{m}} > \mathbf{m}_*)$

@ $T \approx M_1$ thermal density is obtained $\quad \mathbf{n_{N_1}} \sim \mathbf{n}_\gamma$

any asymmetry N1 makes is washed out

@ $T < M_1$ N1's start to decay

@T_α ID $\ell_\alpha + \mathbf{\Phi} \rightarrow \mathbf{N_1}$ go out of equilibrium

$$\mathbf{\Gamma_{ID}}(\ell_\alpha \mathbf{\Phi} \rightarrow \mathbf{N_1}) \sim \frac{\mathbf{1}}{\mathbf{2}} \mathbf{\Gamma}_{\alpha\alpha} \mathbf{e}^{-\mathbf{M_1}/\mathbf{T}_\alpha} \sim \mathbf{\Gamma_D} \mathbf{e}^{-\mathbf{M_1}/\mathbf{T}_\alpha} < \mathbf{H}$$

@$T < T_\alpha$ N1's decay out of equilibrium

$$\kappa_\alpha \sim \frac{\mathbf{n_{N_1}}(\mathbf{T}_\alpha)}{\mathbf{n_{N_1}}(\mathbf{T} >> \mathbf{M_1})} \sim \mathbf{e}^{-\mathbf{M_1}/\mathbf{T}_\alpha} \sim \frac{\mathbf{m}_*}{\widetilde{\mathbf{m}}_{\alpha\alpha}} \quad (\widetilde{\mathbf{m}} > \mathbf{m}_*, \quad \widetilde{\mathbf{m}}_{\alpha\alpha} > \mathbf{m}_*)$$

Washout Scenarios

Intermediate Washout Scenario $\Gamma_D > H \quad (\tilde{m} > m_*)$

$\widetilde{m_{\alpha\alpha}} < m_*)$

@ $T \approx M_1$ thermal density is obtained $n_{N_1} \sim n_\gamma$

because of large couplings to other flavors

$y_{\alpha 1}$ is small anti-asymmetry produced $-\epsilon_\alpha n_\gamma$

@ $T < M_1$ N_1's start to decay

asymmetry produced $\epsilon_\alpha n_\gamma$

lowest order lepton asymmetry vanishes

Washout Scenarios

Intermediate Washout Scenario $\Gamma_D > H \quad (\tilde{m} > m_*)$
$\widetilde{m_{\alpha\alpha}} < m_*)$

@ $T \approx M_1$ thermal density is obtained $n_{N_1} \sim n_\gamma$

because of large couplings to other flavors

$y_{\alpha 1}$ is small anti-asymmetry produced $-\epsilon_\alpha n_\gamma$

@ $T < M_1$ N_1's start to decay
asymmetry produced $\epsilon_\alpha n_\gamma$

lowest order lepton asymmetry vanishes

but a small part of anti-asymmetry washed out

before decay $-\dfrac{\widetilde{m_{\alpha\alpha}}}{m_*}\epsilon_\alpha n_\gamma$

$$\kappa_\alpha \sim \frac{\widetilde{m_{\alpha\alpha}}}{m_*} \quad (\tilde{m} > m_*, \quad \tilde{m}_{\alpha\alpha} < m_*)$$

Washout Scenarios

Weak Washout Scenario

$$\mathbf{\Gamma_D < H \quad (\tilde{m} < m_*)}$$
$$\mathbf{\widetilde{m_{\alpha\alpha}} < m_*)}$$

thermal density is not obtained
as production is not efficient @ $T \approx M_1$

$$\mathbf{n_{N_1} \sim \Gamma_{prod} \frac{1}{H} n_\gamma \sim \frac{\tilde{m}}{m_*} n_\gamma}$$

Washout Scenarios

Weak Washout Scenario

$$\Gamma_D < H \quad (\tilde{m} < m_*)$$
$$\widetilde{m_{\alpha\alpha}} < m_*)$$

thermal density is not obtained
as production is not efficient @ $T \approx M_1$

$$n_{N_1} \sim \Gamma_{prod} \frac{1}{H} n_\gamma \sim \frac{\tilde{m}}{m_*} n_\gamma$$

as before

lowest order lepton asymmetry vanishes
but a small part of anti-asymmetry washed out
before decay $\quad -\frac{\widetilde{m_{\alpha\alpha}}}{m_*}\epsilon_\alpha n_{N_1}$

$$(\tilde{m} < m_*, \quad \tilde{m}_{\alpha\alpha} < m_*) \qquad \kappa_\alpha \sim \frac{\widetilde{m_{\alpha\alpha}}\tilde{m}}{m_*^2}$$

Boltzmann Equations

Lepton asymmetry is a result of competition

$$\frac{dn_{N_1}}{dz} = -(D + S)(n_{N_1} - n_{N_1}^{eq})$$

$$z = \frac{M_1}{T}$$

modify N1 abundance

$$n_L = 2(n_\ell - n_{\bar{\ell}})$$

$$\frac{dn_L}{dz} = \epsilon_1 D(n_{N_1} - n_{N_1}^{eq}) - W\, n_{B-L}$$

source of asymmetry

washout term

accounts for decays/ inverse decays

accounts for $|\Delta L|=1$ scattering

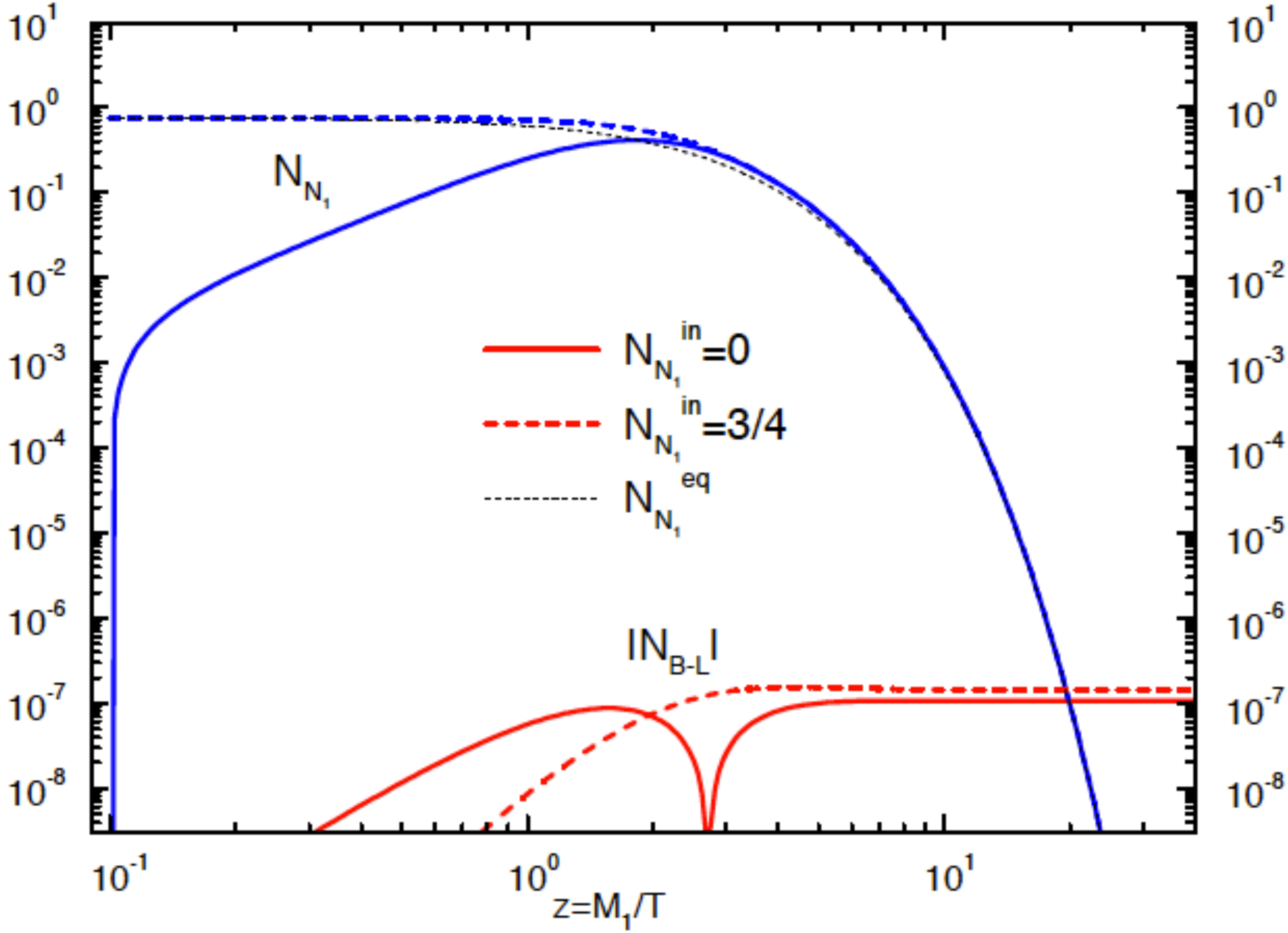
accounts for all processess

$$D = \frac{\Gamma_D}{H z}$$

$$S = \frac{\Gamma_S}{H z}$$

$$W = \frac{\Gamma_W}{H z}$$

Dynamical Generation of n_{N_1} and n_{B-L}



[W. Buchmüller]

Davidson-Ibarra Bound

Casas-Ibarra Parametrization

$$\mathbf{y}_{\alpha \mathbf{i}} = \frac{\mathbf{1}}{\mathbf{v}}(\sqrt{\mathbf{D_M}}\,\mathbf{R}\,\sqrt{\mathbf{D}_\nu}\mathbf{U}^\dagger)_{\alpha \mathbf{i}}$$

$$\mathbf{D_M} = \mathbf{diag}(\mathbf{M_1}, \mathbf{M_2}, \mathbf{M_3})$$

$$\mathbf{D}_\nu = \mathbf{diag}(\mathbf{m}_{\nu_1}, \mathbf{m}_{\nu_2}, \mathbf{m}_{\nu_3})$$

$$\mathbf{R^T}\,\mathbf{R} = \mathbf{R}\,\mathbf{R^T} = \mathbf{I}$$

$$\epsilon_\mathbf{1} = -\frac{\mathbf{3}}{\mathbf{16}\pi}\frac{\mathbf{M_1}}{\mathbf{v^2}}\frac{\sum_\alpha \mathbf{m}_{\nu_\mathbf{i}}\mathbf{Im}(\mathbf{R^2_{1i}})}{\sum_\alpha \mathbf{m}_{\nu_\mathbf{i}}|\mathbf{R_{1i}}|^\mathbf{2}}$$

$$|\epsilon_\mathbf{1}| < \epsilon^\mathbf{DI} = \frac{\mathbf{3}}{\mathbf{16}\pi}\frac{\mathbf{M_1}}{\mathbf{v^2}}\frac{\boldsymbol{\Delta}\mathbf{m^2_{atm}}}{\mathbf{m}_{\nu_\mathbf{1}} + \mathbf{m}_{\nu_\mathbf{3}}}$$

Davidson-Ibarra Bound

$$|\epsilon_1| < \epsilon^{DI} = \frac{3}{16\pi}\frac{M_1}{v^2}\frac{\Delta m^2_{atm}}{m_{\nu_1} + m_{\nu_3}}$$

Requiring

$$Y_{\Delta B}(\infty) > Y^{CMB}_{\Delta B} \sim 10^{-10}$$

$$M_1 \left(\frac{0.1\,eV}{m_{\nu_1} + m_{\nu_3}}\right)\kappa > 10^9 \text{ GeV}$$

$$M_1 > 10^9 \text{ GeV} \quad m_{\nu_1} < 0.1\,eV$$

N1 dominates contribution to leptogenesis

$M_1 << M_2, M_3$ unflavor regime $T > 10^{12}$ GeV

Flavor Effects

Flavor effects can play important role in leptogenesis

Depending on if at the scale of leptogenesis the interactions mediated by charged-leptons Yukawa couplings are in or out of equilibrium

Enter Equilibrium $\Gamma \approx H$

$$\mathbf{y}_\tau^2 \mathbf{T}/(4\pi) \sim \sqrt{\mathbf{g}^*}\mathbf{T}^2/\mathbf{M}_{\mathbf{PL}} \quad \mathbf{T} \sim 10^{12}\ \mathbf{GeV}$$

$$\mathbf{y}_\mu^2 \mathbf{T}/(4\pi) \sim \sqrt{\mathbf{g}^*}\mathbf{T}^2/\mathbf{M}_{\mathbf{PL}} \quad \mathbf{T} \sim 10^{9}\ \mathbf{GeV}$$

@ $T > 10^{12}$ GeV all out of equilibrium: leptons are indistinguishable

For $T < 10^{12}$ GeV flavor effects need to be taken into account

"Single Flavor Approximation" is not valid in this case

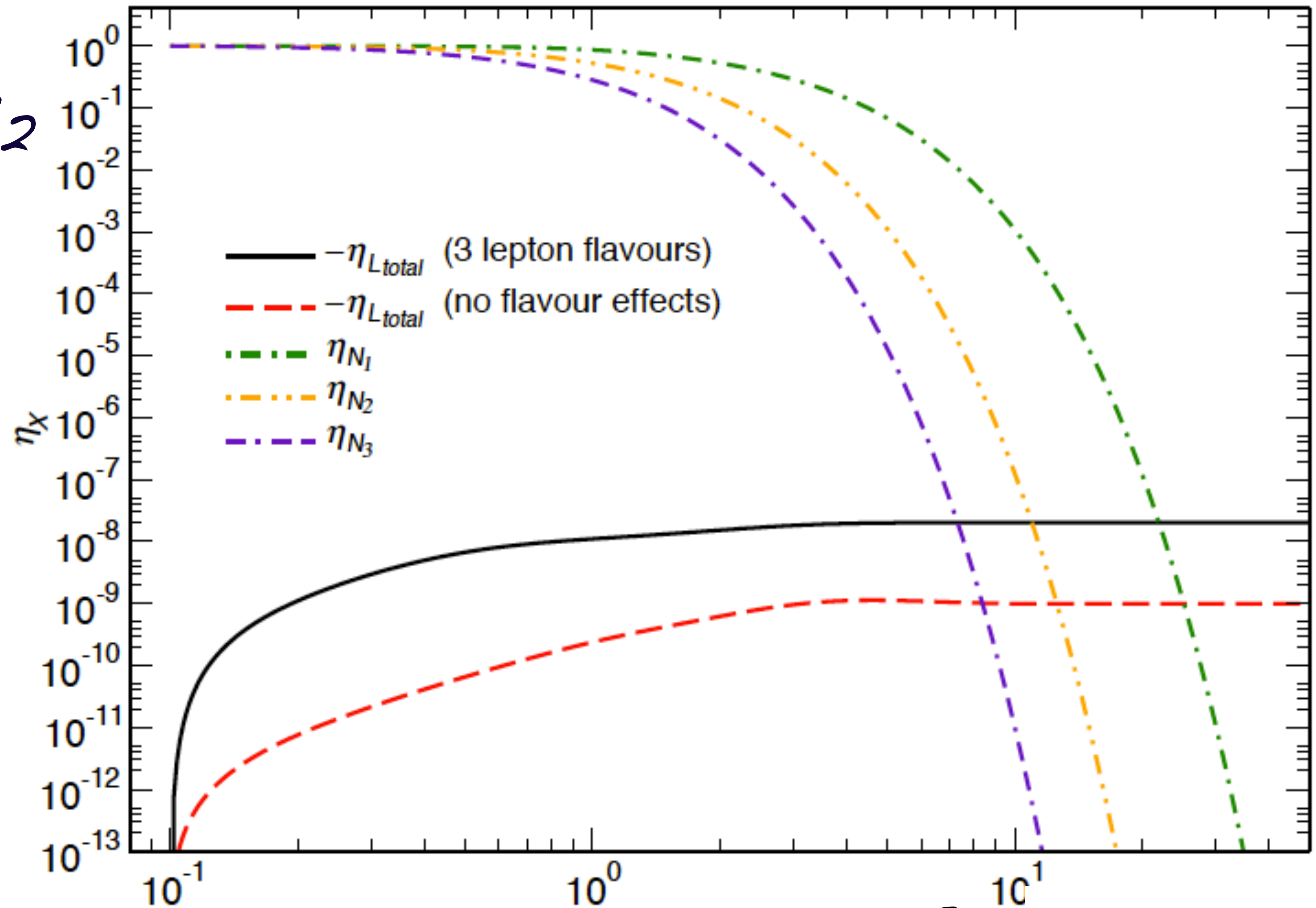
[Abada et al., Nardi et al., Di Bari et al., Antush et al., Pilaftsis etc.]

Heavy Neutrinos Flavor Effects

flavor effects due to N1, N2, N3 Yukawa couplings

$M_2 = 2M_1$

$M_3 = 2M_2$



$z = M_1/T$

[A. Pilaftsis]

Quantum Boltzmann Equations

Fundamental Problem in this treatment:

1. Boltzmann Equations are Classical
2. Collision Terms are $T=0$ S-matrix elements which involve quantum interferences
3. Time evolution should be treated quantum mechanically

New developments: quantum Boltzmann Equations based on Closed-Time-Path formalism [Buchmüller, Di Bari, Plumacher, Strumia etc.]